TOPIC PLAN

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| :---: | :---: | :---: |
| Topic | Function of Several Variables |  |
| Less on title | Partial Derivatives |  |
| Lear ning objec tives | Students will be able to determine partial derivatives of functions of several variables; <br> Students will acquire and deal with derivatives of a function; <br> Students will be able to deal with different problems in everyday life, which require finding partial derivatives of a given function; <br> Students are encouraged to use technology and different software in their work, while considering problem-based situations. | Strategies/Acti vities <br> $\square$ Graphic Organizer <br> Think/Pair/Shar e |
| Aim <br> of <br> the <br> lectu <br> re / <br> Desc <br> riptio <br> n of <br> the <br> pract <br> ical <br> probl <br> em | The aim of the lecture is to make students able to calculate partial derivatives of a function and apply the derivatives to calculate approximation of functions. | -Collaborative learning $\square$ Discussion questions $\square$ Project based learning $\square$ Problem based learning Assessment for |
| Previ ous know ledge assu med: | - Functions <br> - algebraic equations <br> - differentiating techniques | $\square$ Observations $\square$ Conversation S Work sample $\square$ Conference $\square$ Check list $\square$ Diagnostics |

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| Intro |
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Consider the function $f$ given by

$$
z=f(x, y)=x^{2} y^{3}+x y+4 y^{2}
$$

Suppose for the moment that we fix $y$ at 3 . Then

$$
f(x, 3)=x^{2} 3^{3}+x 3+43^{2}=27 x^{2}+3 x+36
$$

Note that we now have a function of only one variable. Taking the first derivative with respect to x , we have

$$
54 x+3
$$

In general, without replacing $y$ with a specific number, we can consider $y$ fixed. Then $f$ becomes a function of $x$ alone, and we can calculate its derivative with respect to $x$. This derivative is called the partial derivative of f with respect to $x$. Notation for this partial derivative is

$$
\frac{\partial f}{\partial x} \text { or } d_{x}
$$

Now, let's again consider the function

$$
z=f(x, y)=x^{2} y^{3}+x y+4 y^{2}
$$

The color blue indicates the variable $x$ when we fix $y$ and treat it as a constant. The expressions $y$, and are then also treated as constants. We have

$$
\frac{\partial f}{\partial x}=2 x y^{3}+y
$$

Similarly, we find $\frac{\partial f}{\partial y}$ by fixing $x$ (treating it as a constant) and calculating the derivative with respect to $y$. From

$$
z=f(x, y)=x^{2} y^{3}+x y+4 y^{2}
$$

We get

$$
\frac{\partial f}{\partial y}=3 x^{2} y^{2}+x+8 y
$$

A definition of partial derivatives is as follows DEFINITION
For $z=f(x, y)$, the partial derivatives with respect to $x$ and $y$ are

$$
\frac{\partial z}{\partial x}=\lim _{h \rightarrow 0} \frac{f(x+h, y)-f(x, y)}{h}
$$

and

$$
\frac{\partial z}{\partial y}=\lim _{h \rightarrow 0} \frac{f(x, y+h)-f(x, y)}{h}
$$

We can find partial derivatives of functions of any number of variables. Since we can apply the theorems for finding derivatives presented earlier, we will rarely need to use the definition to find a partial derivative.

EXAMPLE 1 For $w=x^{2}-x y+y^{2}+2 y z+z$ find $\frac{\partial w}{\partial x}, \frac{\partial w}{\partial y}, \frac{\partial w}{\partial z}$.
Solution In order to find $\frac{\partial w}{\partial x}$ we regard $x$ as the variable and treat $y$ and $z$ as constants. From

$$
w=x^{2}-x y+y^{2}+2 y z+z
$$

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we get

$$
\frac{\partial w}{\partial x}=2 x-y
$$

To find $\frac{\partial w}{\partial y}$ we regard $y$ as the variable and treat $x$ and $z$ as constants. We get

$$
\frac{\partial w}{\partial y}=-x+2 y+2 z
$$

To find we regard $z$ as the variable and treat $x$ and $y$ as constants. We get

$$
\frac{\partial w}{\partial z}=2 y+1
$$

Students can calculate these derivatives using Mathematica on the following way

* Untitled-1 * - Wolfram Mathematica 10.0

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
$\ln [1]=w\left[x_{-}, y_{-}, z_{-}\right]:=x^{\wedge} 2-x y+y^{\wedge} 2+2 y z+z ;$
$\ln [3]=\mathrm{D}[\mathrm{w}[\mathrm{x}, \mathrm{y}, \mathrm{z}], \mathrm{x}]$
Out[3] $=2 x-y$
$\ln [4]=\mathrm{D}[\mathrm{w}[\mathrm{x}, \mathrm{y}, \mathrm{z}], \mathrm{y}]$
Out[4] $=-x+2 y+2 z$
$\ln [5]=\mathrm{D}[\mathrm{w}[\mathrm{x}, \mathrm{y}, \mathrm{z}], z]$
Out $[5]=1+2 y$

\#

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## Quick Check 1

For $u=x^{2} y^{3} z^{4}$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$.

We will often make use of a simpler notation: $f_{x}$ for the partial derivative of $f$ with respect to $x$ and $f_{y}$ for the partial derivative of $f$ with respect to $y$. Similarly, if $z=f(x, y)$, then $z_{x}$ represents the partial derivative of $z$ with respect to $x$, and $z_{y}$ represents the partial derivative of $z$ with respect to $y$.

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One nice use of tangent planes is they give us a way to approximate a surface near a point. As long as we are near to the point $\left(x_{0}, y_{0}\right)$ then the tangent plane should nearly approximate the function at that point. Because of this we define the linear approximation to be,

$$
L(x, y)=f\left(x_{0}, y_{0}\right)+f_{x}\left(x_{0}, y_{0}\right)\left(x-x_{0}\right)+f_{y}\left(x_{0}, y_{0}\right)\left(y-y_{0}\right)
$$

and as long as we are "near" $\left(x_{0}, y_{0}\right)$ then we should have that,

$$
f(x, y) \approx \mathrm{L}(\mathrm{x}, \mathrm{y})
$$

EXAMIPLE 4 Find the linear approximation to $z=3+\frac{x^{2}}{16}+\frac{y^{2}}{9}$ at $(-4,3)$.

Solution So, we're really asking for the tangent plane so let's find that

$$
\begin{array}{ll}
f_{x}(x, y)=\frac{x}{8} & f_{x}(-4,3)=-\frac{1}{2} \\
f_{y}(x, y)=\frac{2 y}{9} & f_{y}(-4,3)=\frac{2}{3}
\end{array}
$$

The tangent plane, or linear approximation, is then,

$$
L(x, y)=5-\frac{1}{2}(x+4)+\frac{2}{3}(y-3)
$$ まu

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