\begin{tabular}{|c|c|c|}
\hline \multicolumn{3}{|c|}{TOPIC PLAN} \\
\hline Partner organization \& \multicolumn{2}{|l|}{UNS} \\
\hline Topic \& \multicolumn{2}{|l|}{Functions of Several Variables} \\
\hline Lesson title \& \multicolumn{2}{|l|}{Applications of directional derivatives} \\
\hline Learning objectives \& \begin{tabular}{l}
\(\checkmark\) Students will be able to determine directional derivatives of functions of several variables, gradient; \\
\(\checkmark\) Students will acquire and deal with derivatives of a function; \\
\(\checkmark\) Students will be able to deal with different problems in everyday life, which require finding directional derivatives of a given function; \\
\(\checkmark\) Students will learn to use their mobile phones as a helping tool in solving mathematical problems
\end{tabular} \& \begin{tabular}{l}
Strategies/Acti vities \\
\(\square\) Graphic \\
Organizer \\
Think/Pair/Shar e
Modeling
Collaborative learning
Discussion questions
Project based learning
\(\square\) Problem based learning
\end{tabular} \\
\hline Aim of the lecture / Description of the practical problem \& \begin{tabular}{l}
Introducing students to several applications of directional derivatives and the gradient \\
Pratical problem is to find different slopes on the surface, maximal and minimal slopes and isohypses
\end{tabular} \& \begin{tabular}{l}
Assessment for learning
Observations
Conversation s
Work sample \\
\(\square\) Conference

Check list
\end{tabular} \\

\hline Previous knowledge assumed: \& | Basic vector calculus |
| :--- |
| Differential calculus |
| Definition of partial derivatives |
| Calculation of partial derivatives | \& | $\square$ Diagnostics |
| :--- |
| Assessment as learning $\square$ Selfassessment | \\

\hline
\end{tabular}

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Introduction /
Theoretical
basics

In the introduction the notion of directional derivatives Is recalled. After that a couple of basic examples are calculated.

1. Find the gradient directional derivatives of the function

$$
z \mid=x^{3}-x y^{2}
$$

From In the point $A(1,1)$ find the values of partial derivatives and directional derivatives in the direction of
a) Vector $\vec{a}=(3,4)$
b) towards the point $B(-1,2)$.
c) In the direction of Points $\mathrm{C}(1,2(, \mathrm{D}(2,1)$

The students should notice that the solution of c) is the same as the partial derivative values and acknowleging that partial derivaties can be seen as a special directional derivetives.

The the teacher gives the next problem to the students:

1. Suppose that the height of a hill above sea level is given by $z=4-0.1 x^{2}-0.2 y^{2}$. If you are at the point $(60,100)$ in what direction is the elevation changing fastest? What is the maximum rate of change of the elevation at this point?
Afetr solving this problem students are directed towards a 2D and 3D heat map and the notion of isohypse is recalled. Using the surface from the previous problem the natural question of which direction should the mountaineer move in order to stay on the same elevation.


After solving this problem students are divided into groups. They are given a task to visualize the previous problem their mobile phones try to visualize the surface from They
are directed to several websites and Geogebra animations their mobile phones try to visualize the surface from They
are directed to several websites and Geogebra animations that give a nice visualization of the problem.
$\square$ Peerassessment

$\square$ Presentation

$\square$ Graphic Organizer $\square$ Homework

## Assessment of

 learning$\square$ Test
$\square$ Quiz
$\square$ Presentation
Project
$\square$ Published work
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|  |  <br> At the end of the class students compare their solutionsand discuss them among themselves. |
| :---: | :---: |
| Action | Using differentiating techniques and basic vector calculus after viusalising a problem. |
| Materials / equipment / digital tools / software | The materials for learning are given as a part of references of the end from this topic plan; <br> Equipment: classroom, whiteboard, marker in different colours; <br> Digital tools: laptop, projector; <br> Software: Geogebra |


| Consolidatio | With the given examples students can consider that the real functions and their |
| :--- | :--- | n derivatives are important for solving real life problems. Students will learn what is a directional derivative of a function and gradient and how to calculate it. They can learn how to apply directional derivatives in a real problem. Students can use technology, different digital tools and software as a help for solving problems, but can also realize that even with technology, solving different everyday problems is difficult without math knowledge.

Reflections and next steps
Activities that worked
Parts to be revisited
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Problem solving, collaboration, using technology
Depends on the students, in a conversation with students the teacher will realize the difficulties that students had and then revisit appropriate parts.

## References

[1] J. Stewart, Calculus, Thomson Learning, China, 2006.
[2] M. L. Bittinger, D. J. Ellenbogen and S.A. Surgent, "Calculus and its applications", Addison-Wesley, 2012.
[3] T. Došenović, A. Takači, D. Rakić, Udžbenik iz Matematike II za studente Tehnološkog fakulteta, Univerzitet u Novom Sadu, 2017.
[4] www.geogebra.org

