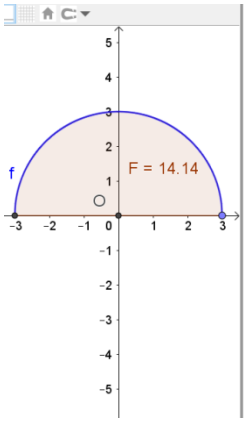
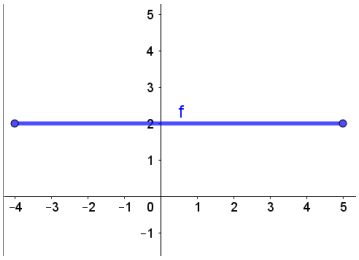
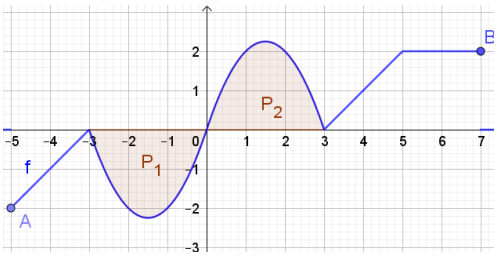
Co-funded by the Erasmus+ Programme of the European Union

TOPIC PLAN		
Partner organization	UNS	
Topic	Definite integral	
Lesson title	**Function defined by integral**	
Learning objectives	Better understanding • definite integral, its definition, properties, and application for determining the area of plane objects. • the examining functions	**Methodology** xModeling ☐Collaborative learning ☐Project based learning xProblem based learning **Strategies/Activities** ☐Graphic Organizer ☐Think/Pair/Share xDiscussion questions
Aim of the lecture / Description of the practical problem	The aim of the lecture is to enable students to examine the functions defined by the integral, using all previous calculus knowledge, with special emphasis on the application of a certain integral. Graphical representation of function defined by integral is the area. Working with functions defined by integral students have simultaneously to work with its multiple representations, algebraic, graphic and verbal. Therefore it is STEAM approach to bended learning of definite integral and its application in dynamic softer neighborhood.	
Previous knowledge assumed:	Derivatives and antiderivaties, their calculations and applications Definite integral, definition, properties and applications	**Assessment for learning** xObservations xConversations xWork sample ☐Conference ☐Check list ☐Diagnostics
Introduction / Theoretical basics	Let the continuous function f is given on the interval $[a,b]$. Funcija F data sa $$F(t)=\int_c^t f(x)dx,\ \ c\in[a,b].$$	**Assessment as learning** ☐Self-assessment ☐Peer-assessment ☐Presentation ☐Graphic Organizer xHomework
	 $f(x)=\sqrt{9-x^2},\quad x\in[0,3],$ $F(t)=\int_{-3}^{t} f(x)dx$ $F(3)=\int_{-3}^{3} f(x)dx=\frac{9\pi}{2}\approx 14.14$ $F(3)=\frac{x\sqrt{9-x^2}}{2}+\frac{9\arcsin(x/3)}{2}\Big\vert_{-3}^{3}=9\arcsin 1=\frac{9\pi}{2}$ $F(0)=\int_{-3}^{0} f(x)dx=\frac{9\pi}{4}\approx 7.07$ $F(3)-F(0)=\int_{0}^{3} f(x)dx=\frac{9\pi}{4}\approx 7.07$	**Assessment of learning**

	On the previous Figure drawn by *GeoGebra*, the function $f(x) = \sqrt{9 - x^2},\ x \in [-3,3]$ and its graph (half circle) simultaneously are consider. The function F is defined as: $$F(t) = \int_{-3}^{t} f(x)dx,$$ The value $F(3)$ is equal to area of half circle. It is calculated, by *GeoGebra* as the integral of function f, from *-3* to *3*. The usual calculation of definite integral, of course presented the same result. **The properties** of the functions defined by integral: 1. Function F is continuous over its domain. 2. $\mathrm{F(c)} = 0$. 3. $F'(t) = f(t)$. 4. $F''(t) = f'(t)$	xTest ☐Quiz ☐Presentation ☐Project ☐Published work
Action	Questions to students: 1) The function f is given on the interval $[-4,5]$ by its graph  The function F is given by integral $$F(t) = \int_{0}^{t} f(x)dx, \quad t \in [-4,5]$$ a) Fulfill the table t | -4 | -2 | 0 | 3 | 5 $F(t)$ | | | | | b) Determine the interval where $F(t) > 0$. c) Determine the interval where $F(t) < 0$. d) Determine the interval where F is increasing? e) Determine the interval where F is decreasing f) Determine an algebraic representation for f. g) Determine an algebraic representation for F. h) Determine domain of F i) Determine range of F j) Determine F'.	

k) Determine F''.

2) The function f is given on the interval $[-5,7]$ by its graph



The function F is given by integral

$$F(t) = \int_0^t f(x)dx, \quad t \in [-5,7]$$

If $P_1 = P_2$ then

a) Fulfill the table

t	-5	-3	0	3	5	7
$F(t)$						

b) Determine the interval where $F(t) > 0$.
c) Determine the interval where $F(t) < 0$.
d) Determine the interval where F is increasing?
e) Determine the interval where F is decreasing
f) Determine an algebraic representation for f.
g) Determine an algebraic representation for F.
h) Determine domain of F
i) Determine range of F
j) Determine F'.
k) Determine F''.

Materials / equipment / digital tools / software

The materials are given in the references at the end from this topic plan;
Equipment: classroom, white or green board;
Digital tools: laptop, projector;
Software: *GeoGebra*, used for multiple representation of presented object

Consolidation	• The teachers and the students use: teaching materials, equipment, digital tools, *GeoGebra* software; • The teacher's and students' discussion about the cognitive conflicts that appear (positive and negative valus of definite integral and its application; • Independent solving of simple tasks by the students under the supervision of the teacher; • Given of examples by the teacher for introducing a new concept in a cooperation and a discussion with the students; • Assignment of homework by the teacher with a time limit until the next class.

Reflections and next steps

Activities that worked	**Parts to be revisited**
The teacher should fulfilled this part after the class	The definition of definite integral and its application for determining the area, will be revised, after the overview of the students' homework and discussion at the beginning of the next class, in the form that should be necessary.

References

1) Bittinger, M. L., Ellenbogen, D. J., Surgent, S.A., (2012)"Calculus and its applications", Addison-Wesley.
2) Pass, S., (2008) **Curriculum Module: Calculus: Functions Defined by Integrals, Microsoft Word - 08-0625.AP.CurricModCalculusFunctionsDefined080115_JB_HLD.doc (collegeboard.org)**
3) Schmeelk, J., Takaci, D., Takaci, A., (2013) Elementary analysis through examples and exercises, Kluwer, Springer Science & Business Media.
4) Stewart J., (2006) Calculus, Thomson Learning, China.
5) Takači, Dj., Stankov, G., Milanovic, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups,. *Computers and Education, Vol. 82*, 421-431
6) The film Definition of definite integral can be found on the platform **https://cloud.pmf.uns.ac.rs/s/pQXwNsPD3GtcyEZ**