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PROJECT TITLE: **Mathematics of the Future: Understanding and Application of Mathematics with the help of Technology**

Intellectual Output 2: **Development of new STEAM methodology and technology**

Result-IO2: **Proposals for implementing STEAM technologies and methodologies**

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1. Introduction

The education in the past was preparing the students for the challenges of society through knowledge of the relevant science without any significant links between sciences, technology, and the real world. The students were preparing for jobs, which might not find after graduation. Usually, every student after completing their education applies for a job for which he does not have some deep knowledge. Therefore, he is required to spend additional years in training and in an adapting to the work environment and the work tasks while he is become a productive worker in the company. This means extra learning time for the new employee and a waste of time and money for the company in which he works.

Education is now expected to provide hard skill and soft skill to learners to form competent human resources so they can compete internationally. In education, there are some subjects that are important to learn because they have many benefits both in real life and in integration with other subjects. One of the most important subjects to learn in the school is Mathematics. Mathematics is an important lesson because Mathematics is very useful in life and because it teaches essential life skills, such as the ability to carry out arithmetic and an understanding of shapes, angles, and distances. One of the major benefits in comparison to many other academic subjects is that its concepts are also more universal, crossing over to different languages easily, (Milaturrahmah, N. et. al, 2017).

The field of mathematics can also be used to teach students the importance of truth, the value of identifying patterns and trends, the purpose of structure, and the need to use logic and reasoning. These kinds of skills and abilities are transferable to other areas too, and so can be crucial for cognitive development on a more general level. It is also critical to understand that some of the importance of mathematics is connected to its value within the workforce, making it useful for professional development (Milaturrahmah, N. et. al, 2017,2).

Mathematics also can be integrated with various disciplines such as Science, Social Studies, Art, Health, Reading/Language Arts and Physical Education. The 21st century demands competent human resources in Science, Technology, Engineering design and Mathematics. With regard to economic growth of the 21st century, the workforce must have the skills of Science and Mathematics, creativity, expertise in information and communication technology, and the ability to solve complex problems (Milaturrahmah, N. et. al, 2017).

Because of the quality of teaching, students find mathematics very abstract and fear mathematics. Thus, there may be difficulties in transferring the information learned in the classroom to the daily life. Mathematics can be a daunting subject for many students, and it has historically been taught in a traditional way, which does not suit all learning styles. Mathematics is commonly perceived to be very difficult, (Fritz, A. et. al, 2019). This can make it difficult for some students to engage fully, which can lead to a lack of understanding about some of the essential mathematical concepts that later lessons are built upon. Many believe “it is ok—not everyone can be good at math”, (Rattan, A. et. al, 2012). With such perceptions, many students stop studying mathematics soon after it is no longer required of them. Giving up learning mathematics may seem acceptable to those who see mathematics as “optional,” but it is deeply problematic for society as a whole, (Li, Y., Schoenfeld, A. H., 2019).

Mathematics is a gateway to many scientific and technological fields. Leaving it limits students’ opportunities to learn a range of important subjects, thus limiting their future job opportunities and depriving society of a potential pool of quantitatively literate citizens. This situation needs to be changed, especially as we prepare students for the continuously increasing demand for quantitative and computational literacy over the twenty-first century (e.g., Committee on STEM Education 2018), (Li, Y., Schoenfeld, A. H., 2019).

The modern time flow, the development of computers and techniques, as well as the development of society, have opened new demands on science and education in search of new methods for access, learning and acquisition of knowledge among students. A particularly an important element is the link between the education and the real life. On the other side, the Covid-19 pandemic has put additional pressure on educational institutions to apply the computers and the technology of distance. The set of all situations required the formation of a new type of student who would respond to the challenges of the new time. One such method that is increasingly taking place in the educational processes at every level is the STEAM methodology.

The fact that the learning environments are teacher-centered, and uniform can be one of the reasons why students have difficulty in implementing information in their daily lives. In this sense, the subjects taught in the course should be taught by different practices and activities in a way that is more meaningful and related to daily life. One of these activities, mathematical modeling activities, can be said to show the relationship between real life and its applicability to real life, (Kertil, M., Gurel, C., 2016).

One of the ideas that have been recommended to improve learning outcomes is to try to teach mathematics by framing some of the traditional math problems in a more integrated way, which allows students to connect the concepts to real-life situations. This is relatively simple with areas like physics and engineering, where mathematics plays a key role already, but it can also be done with the remaining STEAM education fields as well (https://www.viewsonic.com/library/education/steam-education-the-importance-of-mathematics/).

STEAM (Sciences, Technology, Engineering, Arts a Mathematicsis) is educational framework to relate things to each other and reality. It was introduced by Georgette Yakman (Yakman G., 2006), and it is based on STEM (Sciences, Technology, Engineering, Mathematicsis) (Yakman G., 2006). Each STEAM fields science, technology, engineering, and mathematics education has evolved to formally include elements of the others within their own standards and practices. Since STEM education implies science and technology interpreted by engineering and arts all based on mathematics the letter A in STEAM mean the implication of social studies (not only arts) in STEM system.

STEAM is an educational approach to learning at every level of the education, which uses Science, Technology, Engineering and Mathematics in conducting dialogue, teacher-student discussion, fostering critical thinking and fostering student issues. So, STEAM connects five components: Science, Technology, Engineering and Mathematics as shown in Figure 1.

Figure 1: fifth components at STEAM

(https://www.google.com/search?q=What+is+the+purpose+of+STEAM+education%3F&rlz=1C1GCEA_enMK952MK952&source=lnms&tbm=isch&sa=X&ved=2ahUKEwit_bXgjbjyAhUshf0HHTVXDw4Q_AUoAXoECAEQAw&biw=1366&bih=600#imgrc=r2H-naNcKR3avM)

STEM/STEAM-based program take an integrated approach to learning and teaching, which requires an intentional connection between curriculum learning objectives, standards, assessments, and lesson design and implementation. STEM/STEAM learning applies meaningful math, science, and technology content to solve real-world problems through hands-on learning activities and creative design.

Science, technology, engineering, and mathematics (STEM) education with technology age is appeared in the twenty-first century; it plays an important role in shaping cultural and economic development, embracing innovation, caring about creativity and problem-solving (Tezer, M. 2019, Cooper, R., Heaverlo, C., 2013). Due to the benefits of STEM education on the development of countries, intensive efforts are being made to reach the desired level between STEM and science education (Tezer, M. 2019, Heaverlo, C., 2013).

Using this educational approach, we got students who think critically, integrate knowledge and skills from multiple areas, get creative ideas to apply the acquired knowledge in solving real problems. From them, the society later gets innovators, leaders, educators, i.e., creative people who will lead the society forward in development and progress.

STEAM is also defined as an integral learning approach that requires a link between standards, assessments, and lesson design / implementation. In reality, STEAM requires and includes two or more standards of its five components that should be taught in the classroom, but also to be evaluated with each other.

For instance, mathematics has increasingly become integrated with technology through things like computer databases and spreadsheets. In fact, computers have actually played a key role in proving that some long-standing mathematical concepts hold true. Moreover, mathematics can also be integrated with the liberal arts, with psychology, in particular, being a good example of this, as it is a subject area that utilizes statistics and data samples (https://www.viewsonic.com/library/education/steam-education-the-importance-of-mathematics/). We can say that the goal of STEAM-based learning provides students with access to tools and methods that pave the way for research of new and creative ways to solve real-world problems, work with data, learn about innovation, and connect between themselves in the fields that are of interest to them. In fact, STEAM teaches the students to think more broadly than what they get while studying. The benefits of learning with this methodology are illustrated in Figure 2.

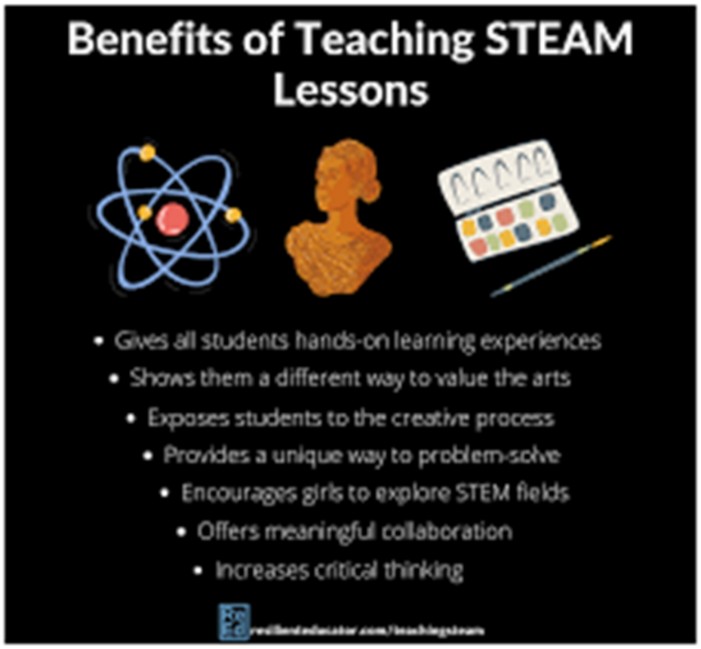

Figure 2: Benefits of learning STEAM lessons

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In fact, STEAM-learning is clearly described in the following Figure 3.

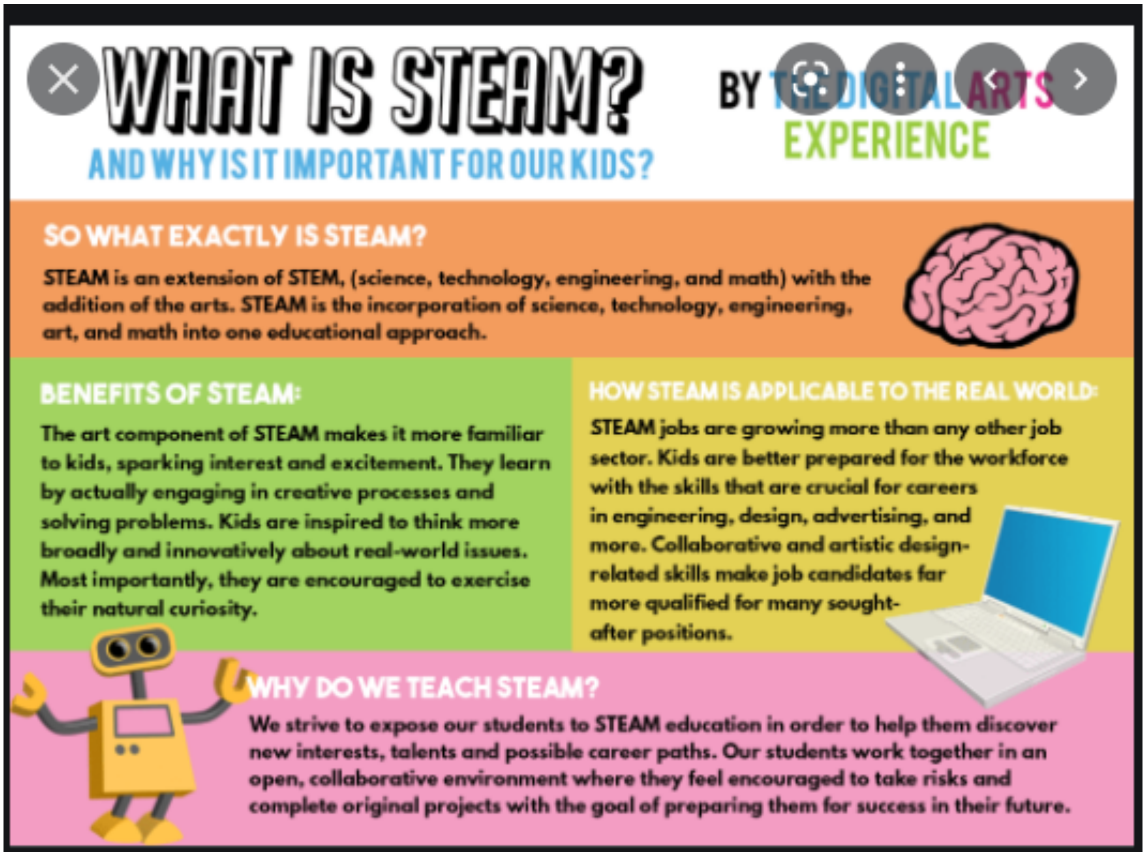

Figure 3: STEAM-learning

(https://www.google.com/search?q=What+is+the+purpose+of+STEAM+education%3F&rlz=1C1GCEA_enMK952MK952&source=lnms&tbm=isch&sa=X&ved=2ahUKEwit_bXgjbjyAhUshf0HHTVXDw4Q_AUoAXoECAEQAw&biw=1366&bih=600#imgrc=LYFn9sDXI12o_M)

In order to compete in the global economic system of the 21st century, a country must establish an education where students gain an understanding of Science, Mathematics, Engineering and computer (Technology), and produce the product using the skills required in the field, (Devrim, A., 2016). Furthermore, the recent Science and Technology education is a constructivism and investigation aimed at integration with other disciplines such as the use of effective technology (Engineering) and problem-solving skills (Mathematics) in building the foundation of STEM education (Milaturrahmah, N. et. al, 2017).

Mathematics as a subject is often mentioned as underpinning the other disciplines of STEM because it serves as a language for science, engineering, and technology (Schmidt, W. H., 2007, Fitzallen,

N., 2015).
Stating that mathematics underpins the other disciplines sets mathematics up in a supporting role in integrative STEM education contexts. Ideally, mathematics should be given more standing and be considered an enabler or imperative for the advancement of understanding of concepts in other disciplines, (Fitzallen, N., 2015).
Silk and his colleagues suggest, "One way to do this is to repeatedly foreground" the desired [mathematical] content while temporarily pushing other concepts into the background" A shift in focus from the incidental nature of mathematics in learning activities to a focus on the instrumental nature of the mathematics may be one way of making the mathematics more explicit within STEM learning contexts and activities. Research is required to determine which integrative approaches put mathematics most effectively to the forefront of learning experiences (Fitzallen, N., 2015).
STEM implemented early in school will have a major impact on a country in facing global challenges, as Hidayah and Rohaida say that with the existence of information technology, economy-based knowledge, mastery of science and technology to students, schools must produce human resources who are knowledgeable and competent with the ability and creativity sufficient to lead the developed countries. Through education-based Science, Technology, Engineering and Mathematics (STEM), technological developments can be improved to meet global challenges (Milaturrahmah, N. et. al, 2017).
Biçer, A. (2020) in his paper discuss about a new model of STEM schools known as Inclusive STEM high schools (ISHSs). Unlike the original STEM schools, ISHSs accept students regardless of their previous academic achievement in STEM. Schools classified as ISHSs share two primary goals: a) developing all students' mathematics and science achievement, and b) reducing the mathematics and science achievement gap between students who come from traditionally unrepresented subpopulations and students who come from traditionally upper-class demographic groups.

STEAM is a curriculum based on educating students in these five specific areas using an interdisciplinary approach. It involves teachers and students, connecting and pooling different perspectives in search of a better understanding of STEAM. Some of the skills that students are likely to develop, when STEAM is implemented correctly are problem solving, critical thinking, teamwork, independent thinking and learning, communication, collaboration, creatively, selfreflection, critical thinking and most importantly, digital literacy (CcHUB, 2020). What STEAM subjects generally consist of, is given below:

- Science is observing, experimenting, asking questions, wondering how things work, making predictions, and sharing findings. This includes the biology, chemistry, and physics, but also includes subjects such as psychology, geology, and astronomy.
- Technology is being inventive, using a variety of tools, making things work, identifying issues, using computers. Technology includes topics such as computer science, software development, AI, and programming.
- Engineering is problem solving, testing materials, designing, creating, building. The four main areas of engineering are chemical, civil, mechanical, and electrical engineering.
- Art is to recognize, know, use, and demonstrate a variety of appropriate arts elements and principles to produce, review and revise original works in the arts. It is also understanding of how to express ideas and feelings, use words, manipulate tools and media, and solve problems.
- Math is patterning, sequencing, exploring shapes, numbers, volume, and size. Includes learning geometry, fractions, algebra and statistics at school, and math also encompasses subjects related to economics.

STEAM industries are continually growing and changing, and so teachers of STEAM subjects need to take a creative and adaptable approach to their teaching styles to get the most out of their students. In STEAM teaching, you don't simply want to provide information and correct students when they make mistakes. Instead, an ideal STEAM learning environment is all about asking questions and encouraging independent thinking. In STEAM, failure teaches students to problem-solve and is an essential part of growth (FutureLearn, 2021).

STEAM education is an opportunity for students to collaboratively solve engaging and relevant problems using innovation and creativity (West Virginia Board of Education, 2020-2021).

According to West Virginia Board of Education the levels of STEAM integration are:

- Disciplinary - Students learn content separately in disciplines.
- Multidisciplinary - Students reference a common theme while learning content in separate disciplines.

- Interdisciplinary - Students learn content in two or more closely linked disciplines to develop deeper understanding
- Transdisciplinary - Students undertake real-world problems and projects that task students with applying knowledge and skills from many disciplines which shape the learning experience.

STEAM education includes reviews of the epistemologies of general and discipline specific developments in conjunction with the individual discipline's standards, as related to integrative, or holistic, education. Investigating these educational relationships to one another is currently being explored to find the commons of education in relation to pedagogy and language. Along with the development of commons is the need for the disciplines to work with one another in a structure that can be adaptable to the many variations of discipline combinations that make up different directions that people in society pursue. (Picar at al., 2020)

STEAM education is preparing students for the future. According to research in literature review we can say that STEAM education was developing very fast before Covid19 crisis. But the whole education all over the word during Covid19 was based on high technology and therefore it is necessary to work on improving STEAM.

In our research we consider mathematics education connected with sciences, technology, Engineering, and arts education. By using the results of the Intellectual output 1 “Analysis report on state of art in using technology to support teaching in Mathematics after Covid-19 crisis” we prepared methodology the guides for STEAM learning education in blended environment. Blended environment in 21st century during and after Covid19 “combine face-to-face instruction with technology-mediated instruction using information and communication technologies (ICT) mediate the learning experience and interactions without requiring that learners and instructors be located together”. (Graham et al., 2003). Covid 19 crisis forced the students all over the world to apply blended learning.
In this report the guides for STEAM lesson plans are prepared in section 2, which are based on the STEAM methodology and STEAM technology in blended learning environment presented in sections 3, 4, 5. The focus of STEAM teaching and learning is mathematics contents and the introduction of STEAM in mathematics education is presented in section 6.

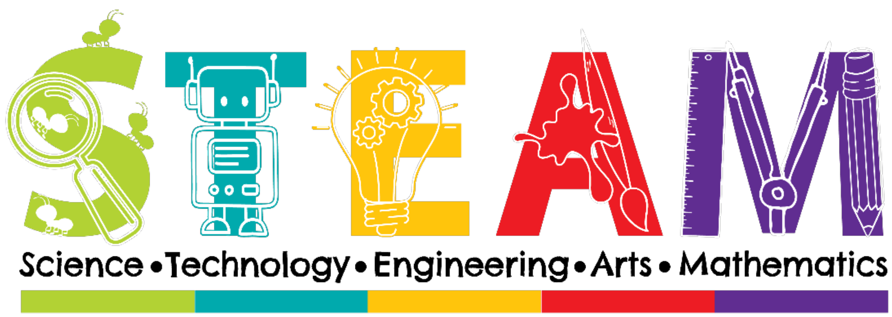

Figure 4 STEAM [http://www.elc-manatee.org/Steam-Machine]

2. Guides for STEAM learning in blended environment

In this section we present the guides for STEAM lessons that can be applied on each course in mathematics, sciences, technology, and arts. These guides are prepared in blended learning environments, because in this 2021 year the whole educational institution all over the world are applying blended learning. Blended learning has been shown to be of great value and irreplaceable for the whole education. Therefore, it is improving very much and very fast in last two years during under Covid 19 circumstances.

These guides contribute both for distance and face to face teaching and learning and can use for preparing lectures during and after Covid 19 crisis.

2.1 Guides for STEAM lessons

In preparing STEAM lessons it is necessary to take care about the implication of science and technology interpreted by engineering and arts all based on mathematics and real-life situations.

STEAM Education is based on constructivist theories throughout the problem-based learning, project-based learning, collaborative learning, and mathematical modeling (Gross, 2016).

The approach in STEAM-related fields is focused on solving real-world problems. If students can address real problems in their social and environmental context, they will be able to make the connections between STEAM knowledge and its impacts on everyday life. In the STEAM approach, teachers should:

• Generate real life problems to the students or encourage students to identify such problems in their environment, and then guide them to solve those problems (problem-based learning);

• Guide student's work and their projects by using a framework for STEAM design (project-based learning).

The teacher has to rely about the STEAM outcomes (**intentional connections, inquiry based**, **integrity 21st century skills**, **equitable assessment** and **making meaning**) and to prepare teaching material adapted to the technology available in the current situation considering **students' constructivist way of thinking based on:**

1. Mathematical modeling process (Starting from real life, sciences, or engineering situations and problems to obtain corresponding mathematical model, to work mathematically and to interpretive the obtain results)
2. Students Work in small group or individual (2,3,4,5 members small group taking care about forming group process)
3. Project based learning
4. Problem based learning.

While designing a STEAM lesson plan, teachers must consider the main idea of STEAM approaches – connection with real life problems. Several components are the next:

- ✓ The essential thing that emphasizes STEAM lessons is presenting a real-life problem, or some engineering challenge.
- ✓ Next important thing is that students have to be interested to that problem, thus it should be related to their future carrier.

- ✓ STEAM lessons should be student – centered, thus students have the main role in solving given problem.
- ✓ The lesson should integrate science, art and math in solving the problem.
- ✓ The lesson should stimulate students' creativity in solving the problem.
- ✓ The lessons should point the role of technology in solving the problem.
- ✓ Students have to collaborate, communicate, exchange ideas and present their results.
- ✓ Students have to consider the eventual failure as a natural chain in this procedure, and as a step toward designing the successful solution.
- ✓ Students have to be able to design more similar real-life problems.

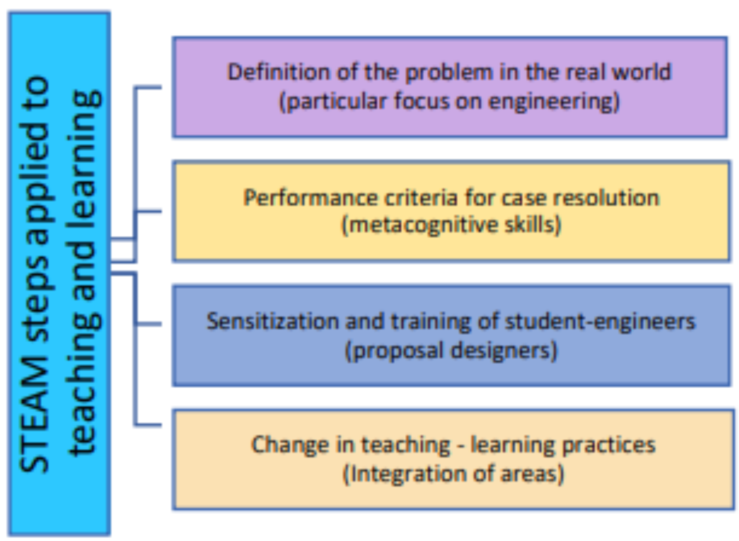

Figure 5: Steps at STEAM methodology

Below we present a more detailed STEAM methodology and STEAM technology to enable teachers to apply the developed guides for STEAM disciplines and adapt their own teaching to each situation they face.

3. How to Use STEAM: Process and Product

There are 6 steps to creating a STEAM-Centered classroom, no matter what area person teaches (The Vision Board, LLC).

1. Focus – this is first step where the question for answer or problem for solving is choose
2. Detail – in this phase we are searching about skills, background information about the defined problem or question
3. Discovery – in this step students and teachers are learning together to overcome difficulties
4. Application – stage where students put their knowledge, analysis, and results into solution
5. Presentation – after creating solution, student must share it and to take feedback about it
6. Link – this step is as loop circle. Students take back feedback about their skills and solution and based on that they can revise their work and produce an even better solution.

4. STEAM methodology

In IO1 it is explain in detail but in the following we present only the main characteristic of STEAM methodology.

4.1 Constructivism

Constructivism is a learning theory, where the learner is an active creator and not a passive recipient of knowledge (Takaci et all, 2015, von Glasersfeld, 1995). According to constructivist theory, the student creates the meaning of new problems, ideas and information, connecting them with his previous knowledge in order to create a product. During learning, the students have to adapt their existing knowledge with the new experience. (Takaci, et all, 2015).

The role of teacher is to form and to adapt the students' environment to enable them learning process to be as much more easily as possible.

- The prepared teachers' different interactive teaching materials including the evaluation and tests for checking students' knowledge should enable students to overcome all cognitive conflicts that may arise during the learning process.
- The teacher should constantly help the students. He coordinates the student activities and

encourages their initiative, encourages student dialogues, suggests that students ask different questions, both to the teacher and to each other. (Takaci, et all, 2015; Doglu et all, 2007). Applying constructivist way of learning the teachers must direct the student to use their experiences that can be questioned or expand upon their previous learning. Teachers must continuously reassure students that they are doing things right, that their thinking has power, and their errors are correctable. Teachers should allow students to choose activities, ask students to explain answers, and prompt all students to be involved.

4.2 Collaborative learning

Collaborative learning is the educational approach where students learn in small groups (two, three, four, members), aimed to solve their problems, to finish the obtained tasks. In collaborative groups the members of the group work together helping one another, exchanging ideas, analyze, find explanations and conclusions that are connected to the common aim to solve the common problem. In literature the authors (Bekele et all, 2011**;** Lin, Huang et all, 2010; and other) agreed that he success of collaborative learning process depends on the forming group procedure In that sense heterogeneous groups (in regard to academic achievement, gender, ethnicity, task orientation, and abilities) are considered as more successful then homogeneous groups (Kagan, 1994; Johnson et all, 1975; Slavin, 1996). According to Kagan (1994), forming heterogeneous groups is a complex problem considering the multiple students' characteristics such as the different level of students' knowledge and their feelings towards one other. During the collaborative learning in a group made up of students of different levels of knowledge, students are introduced to significantly different views of the given problem than their own. Interaction between members of small groups allows students different activities during the learning process and different procedures for solving the same problem. Each student has his own way of learning but when the students learn in the group, they need to harmonize their activities in order to achieve the common goal, the solution of the problem. Some students are asking questions, some of them are answering, some of them are solving problems without any discussions. All these activities can sometimes contribute to different misunderstanding, but at the end they will contribute to the deepening of students' knowledge about the problem.

According to literature (Takaci, et all, 2017; Bekele, et all 2011; Sadeghi, et all, 2016) there are lot of different computer-based methods are used for forming collaborative groups considering multiple students characteristic.

Below we will present one of the possibilities of forming small groups of 4 students according to Kagan, without computers. In literature these small group of four are consider as the best for collaborative work. Before the beginning of collaborative learning, it is necessary for teachers to check the students' previous knowledge of the area to be covered (for example, pre-test). Based on the results of pre-test, a ranking list from the best to the worst student is formed. But, besides the student's previous knowledge the teacher should somehow collect data on students' relationships, their feelings to each other (who wants to work with whom who does not want…). Then, the first small group of 4, is formed by using ranking list. We consider the first, the last and 2 students from the middle to be in the first group. Then we check the students' feelings. If there are no students, among those selected, who have negative feelings towards each other, a group is formed, and the process continues. If there exists at least one such students who have negative feelings towards some student, then he is replaced with the student from the middle of the rang list. When the first group is formed, then the further groups are formed in the same way.

4.3 Mathematical modeling

Since STAEM is the implication of science and technology interpreted by engineering and arts all based on mathematics and real-life situations mathematical modeling is it integrate part. Mathematics is a basis for the sciences, technology, engineering and generally social sciences and mathematical model is simply a representation of the sciences, technology, engineering, and social sciences phenomenon by mathematical equations. This means that one can predict the result of some process from STEAM fields by using mathematical modeling. Understanding STEAM fields phenomenon and mathematics is enough for mathematical modeling. Mathematical modeling is important exercises in mathematics as well as in other STEAM fields. Therefore, mathematical modeling should be a core course in all high school and faculties all over the world. (Gultekin, Kalbekov, 2017).

In recent years, models and modeling has been argued as a means to increase relevance and authenticity in the STEM disciplines (Kertil, M., 2016; Banks, F, 2014; France, B., 2018; Gilbert, J., 2004; Herrington, J. et. al, 2010; Heba, E. D., et. al, 2018; Rau, M. A., 2017). This is not only because modeling is central to the disciplines themselves as authentic practice in laboratories and workshops, but also since modeling is considered a fundamental aspect of STEM instruction. Gilbert, Boulter, and Elmer, (Gilbert, J. K., et. al, 2000) flagged the importance of modeling and models in pursuing an authentic science and technology education, in lucidly stating that:

Authentic' educations in science and technology must reflect the natures of the parent disciplines as far as are practicable. Modeling and models are common to both, thus providing a potential bridge between science education and technology education. The purpose of modeling in both fields is to facilitate communication through a visualization of the relation between the intention and the outcome of the activity, (Gilbert, J. K., et. al, 2000).

As part of the same cogent line of thinking, Davies and Gilbert in (Davies, T., Gilbert, J. 2003) also argued that models and modeling could forge natural links between STEM disciplines such as science and design and technology, due to certain similarities in modeling practices, (Hallström, J., Schönborn, K. J., 2019).

Through processes of modeling in STEM education, the disciplines become bound by a synergistic relationship, often requiring a learner to transit between the learning areas while engaging scientific, mathematical and technological activities, which often render these processes interdependent, (Gilbert, J. K., et. al, 2000).

Mathematical modeling involves a complex process in which a problem state encountered in real life is formulated mathematically and solved with the help of mathematical models, and the solution is interpreted and evaluated in the real world. In this process, mathematics is used to represent, analyze, predict, or otherwise make sense of real-world situations. In mathematical modeling, the individual tries to create a mathematical model that will solve the problem that he/she encounters in real life or in the future. The model in question includes not only mathematical structures but also estimates, assumptions, and strategies for solution. In other words, the solution plan including the assumptions, estimations, and mathematical tools used to solve the problem is the mathematical model for the problem. In addition to being mathematically correct, the model should be meaningful and adaptable for real life. While solving the problem, the individual should also evaluate the meaning of the solution for the real world. All these processes and all the stages of problem-solving in addition to the individual model are mathematical modeling (Hallström, J., Schönborn, K. J., 2019; Pollak, H. O., 2011).

One of the most important tools for transition to STEM education is mathematical modeling (Hallström, J., Schönborn, K. J., 2019).

As mathematical modeling involves activities such as describing natural phenomena or designing a component or a system by writing mathematical equation was mentioned as a component to interconnecting STEM's discipline. Therefore, the ability on constructing a mathematical model in STEM integration focuses should be made, and these value-added had to focus on the existing standard STEM integration practicing. From the philosophical point of view curriculum, model and module of teaching and learning are usually developed based on one or by combination of some of the educational theories on what students should achieve and how they are going to achieve in their teaching and learning. In fact, many theories could explain how students think, act and set strategies for solving the problem (Bajuri, M. R. et. al, 2018; Baumann, C. B et. al, 2010).

The mathematical modeling cycle commonly used in literature is developed by Blum and Leiß (Blum, W., Leiss, D., 2007). In this model, a distinction is made between the real world and mathematics. A prerequisite for this model is that students should understand the mathematical problem and ensure that the model is developed in the real context. It is important to keep in mind that the modeling process is in a repetitive natural loop. (Figure 6)

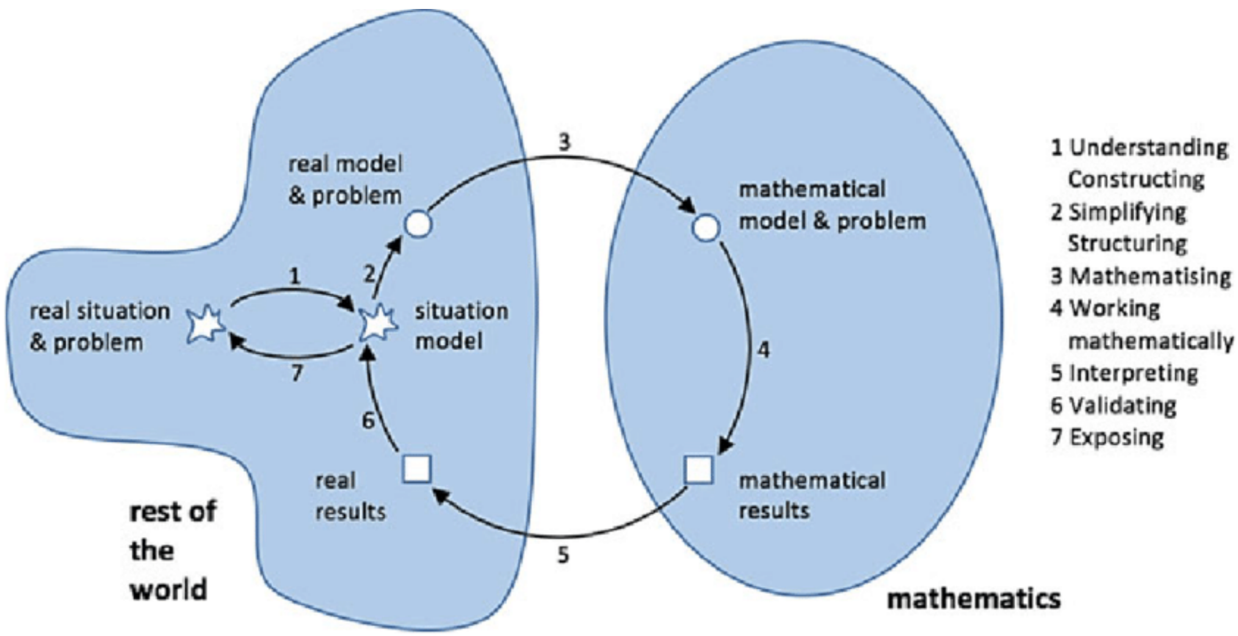

Figure 6. Blum and Leiß [28] modeling cycle.

Another important element is the existence of basic questions arising from the real-world problem in the mathematical cycle. These key questions can help to solve and study a mathematical modeling activity. Key questions are also very important for solving the problem. Another feature of a key issue is that it allows people to focus on the issue. It can also bring people closer to their jobs or problems. In the modeling that Perrenet and Zwaneveld describe (Perrenet, J., Zwaneveld, B., 2012), being outspoken and written communication are of paramount importance. For example, students can conduct a mathematical modeling study and elaborate their solutions. Students also need to think through the modeling process so that they can clearly explain how well they understand the subject after a certain mathematical use. Thus, this mathematical cycle is repeated in a natural way. The revised solution is required during each cycle. This allows students to progress in different ways throughout the modeling cycle before developing an adequate solution. For the realization of such a process, they argue that the mathematical modeling activities of Perrenet and Zwaneveld must be open-ended (Figure 7), (Tezer, M., 2019).

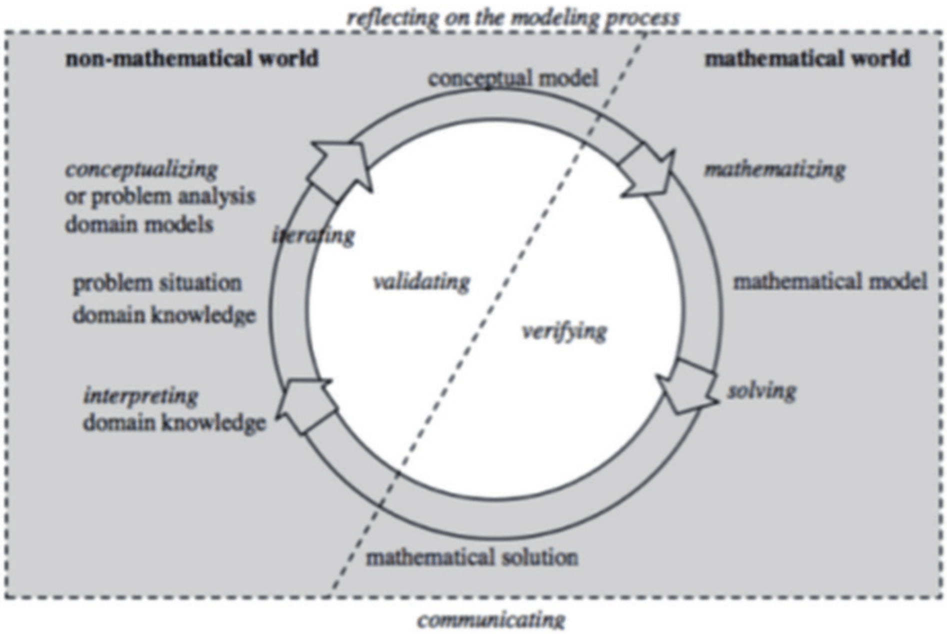

Figure 7. Perrenet and Zwaneveld's modeling cycle (Perrenet, J., Zwaneveld, B., 2012).

Stohlmann and Albarracin, (Doğan, M. F., et. al, 2018) stated that there should be seven items in a mathematical modeling:

- The first one is that the problem should start with a real-world problem.
- Second, key questions should be addressed.
- The third one of these basic elements is the logical thinking of the solution of the problem with mathematical assumptions and approximations.
- Fourth, the mathematics used must be related to the real situation.
- The fifth of these elements plays an important role in written communication.
- The sixth, which is the mathematical modeling process, is an iterative process with open-ended problems.
- The seventh and last item is the reflections in the mathematical modeling used.

However, the most widely used curricula are the modeling activities based on the models and modeling perspective.

Almost all authors dealing with mathematical modeling have similar phases and analyze analog transitions from phase to phase. We from the University of Novi Sad have a good cooperation with Gloria Stillman and apply mathematical modeling according to her scheme. Therefore, we suggest teachers who prepare teaching material in STEAM sense to follow the process of mathematical modeling according to Gloria Stillman. At the beginning, it is important for teacher to choose appropriate problem from STEAM fields, to enable students' understanding, structuring, simplifying and interpreting context. Then after assuming, formulating, mathematising the students must work mathematically and to interpret mathematical output. At the end comparing, critiquing, validating, and justifying are very important cognitive activities too. They should enable students to understand all phases in the process of mathematical modeling, especially mathematical model, and its purpose.

Model-eliciting activities (MEA) are mathematical modeling applications. Mathematical modeling applications are composed of concepts related to different disciplines by their nature. There is not a single definition of mathematical modeling agreed in the literature. Instead, there are definitions, explanations, or shared assumptions made by individual authors. According to Kaiser (Kaiser, G., 2017), mathematical modeling is seen as a creative process to interpret the results and make changes to the model in order to define, control, or optimize the situation in order to make the real-life situation meaningful. One of the many challenges faced by educators is the ways in which complex solutions to unusual problems can be taught to the student in the context of STEM education. One of the tools for transition to STEM training is the MEA, (Tezer, M., 2019).

MEAs, which are integrated into curricula for students to solve complex and difficult real-life problems, force students to build models and encourage them to test their established models, and their theoretical structure is known as a kind of open-ended problem-solving activities based on mathematical modeling perspective. In school mathematics, MEAs have the potential to allow students to use mathematics in a flexible, creative, and powerful way in the STEM field because MEAs support the development of mathematical literacy, productive trends in mathematics (Stohlmann, M. S., et. al, 2013), and a deep and integrated understanding of mathematical content and practices. In MEAs, students clearly document their thought processes, consider their limitations, and use science and mathematics knowledge in the solution of the problem. MEAs offer students the opportunity to work on complex real-life problems involving model development. A framework for quality STEM integration curriculum is linked to the structure of MEAs, (Tezer, M., 2019; Stohlmann, M. S., et. al, 2013).

According to the framework, the curricula

(a) will serve a meaningful purpose and an engaging context,

(b) enable students to develop problem-solving skills and engineering designs,

(c) allow students to have the opportunity to redesign and learn if they fail,

(e) support student-centered pedagogy, facilitator, and cooperative learning, including teacher, and

(f) are designed to promote communication skills and teamwork (Maiorca, C., Stohlmann, M. S., 2016).

The ability to solve a problem in mathematical modeling is closely related to the cognitive activities that are applied while they are facing a problem-solving task. Therefore, good cognitive skills will lead a person to be more analytically minded while facing mathematical modeling problem solving. Consequently, a lot of mathematical modeling activities should focus on cognitive aspects, and this could help students' exposure indirectly to STEM-related careers in real life. There some cognitive theory proposes the ability to set thinking strategy. Mathematical modeling is considered a challenging task and it involves high-level problem-solving abilities and it is proved as an enjoyable task for students to develop their cognitive abilities. Therefore, implementing this task, could lead students to be more analytical as required in STEM careers industry. However, the difficulties of mathematical modeling activities are because students do not know how to regulate their cognitive ability. The cognitive development at this stage is placed under the zone of proximal development, where students need the elements of scaffolding as a means to assist metacognitive activities, (Bajuri, M. R., et. al, 2018).

Real-life problems are discussed in mathematical modeling activities. Real-life situations are complex and cover many areas. Therefore, mathematical modeling is suitable for the different disciplines, and it is seen as an effective tool that can be used in STEM education. This type of activity, which is defined as inter-disciplinary mathematical modeling (IMM), includes an understanding of different disciplines. In the understanding of IMM and in the solution of the problems of real-life situation, one or several disciplines are used together with mathematics. In their study IMM is dealt with in mathematics and science. Therefore, IMM activities represent the activities associated with mathematics and science (Tezer, M., 2019, Doğan, M. F., et. al, 2018).

The IMM process is a cyclical and cascading process. However, unlike mathematical modeling, the inclusion of more than one area of IMM activities leads to a differentiation in the modeling process. The IMM process begins in the real world, and first of all the individual needs to understand the real-life problem. The first step, which is expressed as an understanding of the problem, enters the STEM world (Tezer, M., 2019).

If a good STEM integration is to be implemented, elements of the metacognitive and Vygotsky's social development (Vygotsky, L., 1978) should also be included. As shown in Figure 8, STEM integration can be facilitated if the instructors implement these selected theories. The selected theories could guide this exploration in the STEM integration practitioner. These two theories are proposed based on the following principles so that the practice can be carried out (Tezer, M., 2019).

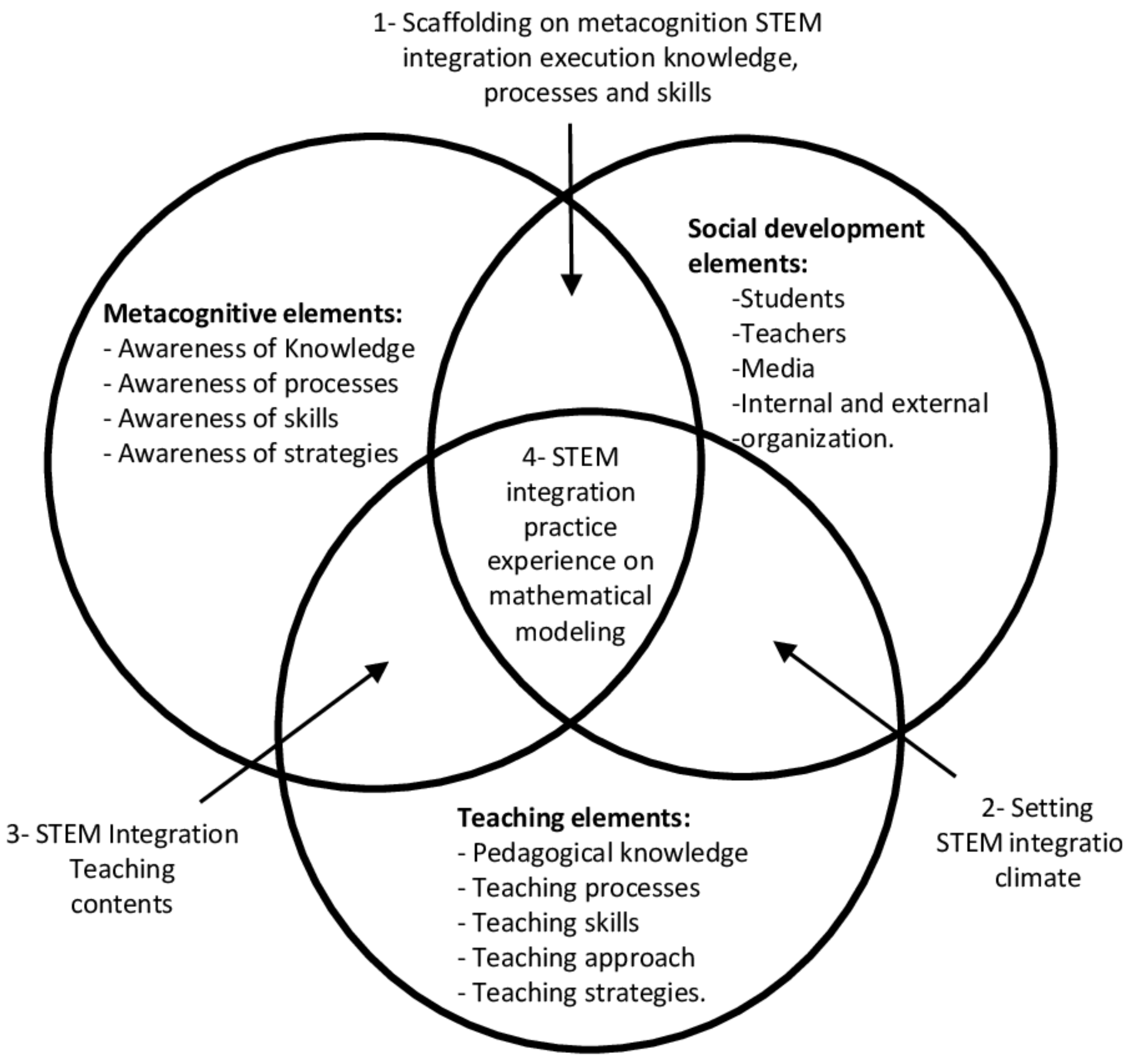

Figure 8. The theoretical framework on metacognition of STEM integration from mathematical modeling perspectives (Tezer, M., 2019).

When Piaget (Piaget, J., 2003), who explained the theory of cognitive development, explained only the characteristics of the cognitive development age stages, he mentioned the best level of learning and the importance of age for thinking development. Thus, Flavell's (Flavell, J. H., 1979) theory (metacognitive knowledge, metacognitive experience, and metacognitive strategies) can explain the students' thinking, strategies, and actions to solve mathematical modeling problems. The problem-solving model of Polya (Polya, G., 2004) may not be sufficient for a STEM practitioner (Proust, J., 2013).

This is because the problem is defined here in only three ways. In particular, supporting mathematical and quantitative processes in science, mathematics, and engineering, and thus increasing mathematical reasoning, is the main objective. Technology provides tools to perform quantitative calculations more efficiently or to produce alternative visualization tools for experimental outputs. All modeling processes share the standard features shown in Figure 9. It has been demonstrated that there is a capability of researching modeling techniques, mathematical reasoning to model engineering design, and the ability to make scientific inquiry and then produce a structure. Mathematical modeling is of particular importance because it is important to produce appropriate tools to predict how quantification methods, new designs, and new situations will behave (Sokolowski, A., 2018).

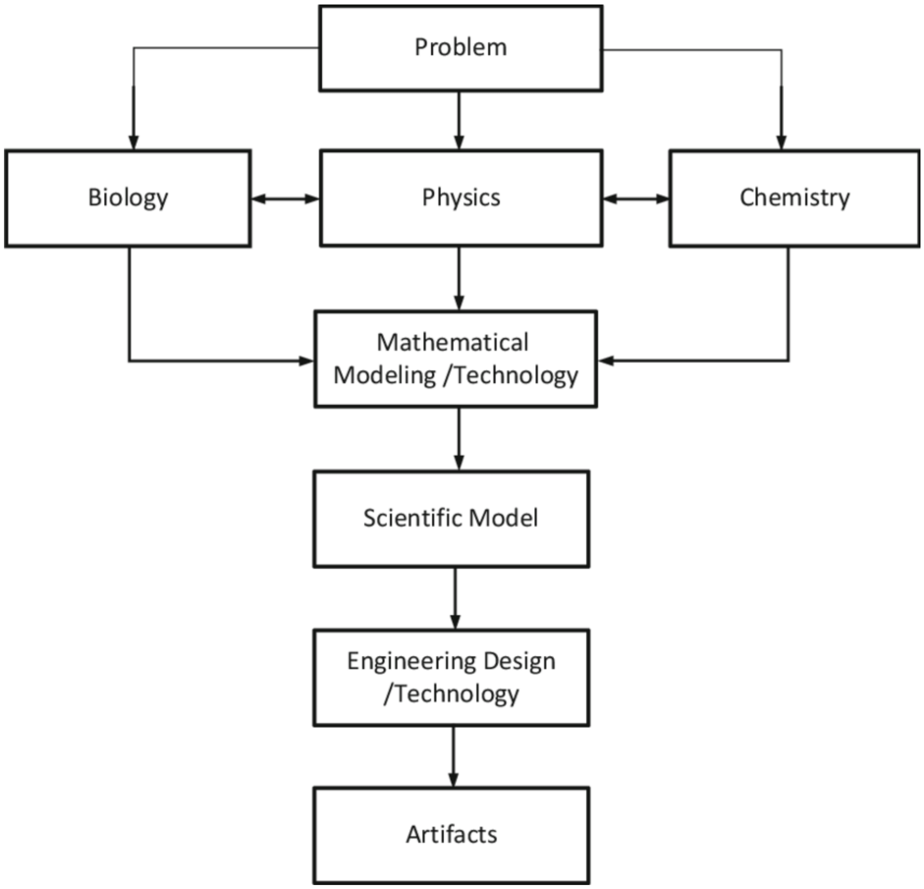

Figure 9. Phases of STEM projects, (Tezer, M., 2019).

A number of different models use, and modeling processes and skills are proposed by the contributions referred to STEM education. But the following three assertions about the role and functions of models and modeling for pursuing authentic STEM education are mainly considered (Hallström, J., Schönborn, K. J., 2019):

First, the STEM community requires a definition and classification of the nature of model types and model uses and components of the modeling processes. Although models and modeling differ slightly between the STEM disciplines, there are some clear similarities, for example, concerning visual models and representations (Tang, K.-S., Williams, P. J., 2018). Modeling is often about representing simplified versions of reality that take on concrete/physical, conceptual, verbal, gestural or symbolic/mathematical forms. Models are therefore simplified representations of phenomena that often include concrete entities that can be smaller or larger than the represented phenomenon. Models could also be abstractions such as force depictions or graphs or equations. Models are therefore representations of ideas, objects, systems, events, or processes which are central in science, technology, engineering and/or mathematics. At a conceptual level, models are even systems of description in themselves, for explaining, constructing, modifying, manipulating and/or predicting a complex series of experiences. Models thereby help to organize relevant information so as to generate or (re)interpret hypotheses about given situations, events, designs or processes, or explain how information is related.

Second, the STEM community requires a classification of central functions of modeling processes and skills associated with models and modeling. Central functions of models are to support development of theories and artefacts through manipulation (e.g., concrete models) or mental exploration (e.g., conceptual models, sketches), and, in the latter case, modeling ideas in the mind for communicating with oneself and modeling ideas in the world for communicating with others (Davies, T., Gilbert, J., 2003). Some of the primary skills associated with modeling include understanding what a model is and how to use it; carefully defining the context of the modeling process (i.e. is it a real-life or an educational context?); mentally visualizing a model outcome; deciding what mode of representation (e.g. physical, visual, verbal or symbolic/mathematical) to express the model in; and understanding how a model can be constructed, interpreted, tested/evaluated, revised and (re-)used. A crucial skill is also being able to evaluate the scope and limitations of a certain model. Nia and de Vries in (Nia, M. G., de Vries, M. J., 2017) further claim that students should learn about the relationship between the intrinsic and intentional/functional nature of models by taking both users' and designers' viewpoints into account:

- Users' view: Associated with understanding how a specific property of the model at hand makes it suitable for serving certain action(s). This understanding can occur in diverse ways such as direct learning about, reflecting upon, or testing ready-made models.

- Designers' view: Associated with how designers learn to produce useful models that realize their intended functions. To achieve such learning, students can be faced with various predefined functions regarding a model and be asked to develop their own, what they consider to be, relevant models.

In such authentic design situations students will also “design” scientific and mathematical formulae and models to optimize their designs; new knowledge is developed about the design process itself but also about science and engineering, wherein students are required to apply existing knowledge previously learnt in science, technology and/or mathematics (Kertil, M., Gurel, C., 2016; Ammon, S., 2017).

Third, common to all four STEM disciplines is the fact that a model can exist in different modes of representation. The ability of students to switch and transit between various representational modes increases their potential for learning, not only of modeling itself but also about the central concepts and practices of the STEM discipline in question. Furthermore, knowledge about mode of representation also increases the potential of using the same model but in different contexts, which also augments the opportunity for interdisciplinary cooperation between the STEM disciplines.

Considering the modeling used in the mathematical modeling process, the emergence of different frameworks and approaches reveals the complex structure of the process. For this reason, it is seen that the studies related to the mathematical modeling process, taking into account the different effects of technology, are combined with STEM, and this leads to the emergence of richer cognitive and metacognitive processes.

As a result of the importance of STEM activities in solving mathematical modeling and real-life problems in different disciplines, STEM activities continue to be integrated into schools. While many countries have added STEM to their education programs, some of them have been combining mathematical modeling with practices. Even teacher trainings on this subject are continuing.

As a result, it can be said that the teaching done by using mathematical modeling together with STEM increases the students’ motivation toward the lesson; they learn better by concentrating their attention on the subject, leaving a positive effect on them; and the students’ success and attitudes toward the lesson increase. Solving real-life problems in the future through STEM and mathematical modeling will continue to play an important role in providing innovative and creative problem-solving perspectives in the cultural and economic development of the countries, (Tezer, M., 2019).

4.4 Project based learning

Project based learning is often referred to as "21st century teaching". According to Vilotijević (2007), it is defined as a learning process based on independent activity of students in the extended time to achieve their goal, and the obtained results are presented in written form. The realization of the project depends on students, on their creativity, fantasy, critical thinking, intrinsic motivation, interests, and requirements. This type of methodology includes students as main part in educational process. All the projects from start to finish are executed by the student and teachers only give ideas for projects. Teachers are included as a person who facilitate the process of teaching and person that help students and encourage them to learn themselves. There is a thinking that this methodology is not cost-effective because it requires space and resources for the projects to be carried out and that this method cannot include all the topics of learning in the various STEAM subjects (CcHUB, 2020). For example, PBL may involve a long-term project that takes place over several weeks or it may take place within one class period. Some PBL tasks or projects involve fabricated problems, while others focus on a problem that exists in the real world.

The process of mathematical modeling is mainly realized through project-based learning. The topics of the project are mostly taken from STEAM fields and should be related to the mathematical content.

Project based learning is constructivist way of learning.

The teacher in preparing teaching material is supposed to take care about the basic stages (phases) of project-based learning according to Vilotijević, (2007):

• Project initiative

• Determining a topic for the project

• Formulation of the project goal

• Project planning

• Project elaboration

• Project implementation

• Project presentation

• Reflection on the project.

It is important for teachers to formulate the goal of project-based learning. For that purpose, he must answer the following questions (Vilotijević, 2007):

• What do we want to achieve?

• In what way, how will we achieve this?

• What is the correlation with the teaching content?

• What are the forms of work?

• What are the sources of information?

• How long will the project be completed?

4.5 Problem based learning

One of the most well-known approaches to active learning in mathematics is a problem-based learning. According to Vilotijević (2007), problem-based learning is based on independent research activity through which students, overcoming problem difficulties, finds new solutions and adopts new truths. This method of teaching requires a high level of thinking. Here the students analyze, create, and evaluate a problem posed. This kind of method encourages students to ask questions and work together in groups (teamwork), not individually. The group of students could be interdisciplinary cutting across various fields in the sciences all working together to proffer a solution (CcHUB, 2020).

Problem solving is fundamental for STEAM approach, but this challenge is very often difficult for the students. Thus, teachers should give them as many directions for working as possible, permanently supervise their work and support them when necessary.

Different digital tools, software and all the achievements of the technology that are accessible to the students should have allowed them to use, because the technology really simplifies problem – solving. Furthermore, students should be encouraged to use it.

Figure 10.
http://www.stem-by-design.com/stem-problems-students-can-really-address/ (downloaded 15.10.2021)

However, when choosing the problem to be solved, teachers should be sure that the level of the problem is appropriate for students' age and the solution can be reached with steps appropriate for them. Teachers should consider what students have learned previously that can help in solving the problem, and do the necessary resources are available to them.

Thus, science, technology, engineering, art and math should all be combined in order the STEAM goals to be reached. Teaching only math can result making students not be able to recognize when and where they can apply the math knowledge they have achieved. Authentic STEAM lessons are designed to integrate maths and science learning content. Measuring, calculating, testing, gathering and analysing data – when students are applying these skills to solving a problem, they can see the maths and science in what they are doing.

For this purpose, teachers should collaborate with other math, science and art teachers to gain STEAM goals and incorporate curriculum objectives into a STEAM lesson. Teachers should also create inter-disciplinary projects sharing their expertise and appropriate resources. Teachers should use math in the context of solving problem in order students to realize the importance of pure math achievements and possibilities to apply them in different environmental situations. These elements create STEAM teaching approaches in the educational process.

On https://elearning.tki.org.nz/Teaching/Future-focused-learning/STEM-STEAM#js-tabcontainer-1-tab-2 (15.10.2021) there are mentioned some learning approaches widely used as frameworks for guiding students through STEAM learning:

- Design Thinking is a design methodology that provides a solution-based approach to solving problems. It contains several different phases, including empathising, defining, ideating, prototyping, and testing.
- The Engineering Design Process (EDP) is a step-by-step method of solving a problem by creating something tangible with a specific function.
- Project-based learning: STEAM learning fits in really nicely with the goals and aims of project-based learning. Perhaps the only difference is that STEAM has an engineering focus.
- Computational thinking enables a student to express problems, and formulate solutions in a way that means a computer (an information processing agent) can be used to solve them.
- Makerspaces - a more relaxed and open-ended version of STEAM, Makerspaces are places where students can follow any kind of interest that involves making, creating, tinkering, programming, and designing.

Problem and project-based learning are not the same. Problem based learning is included in project-based learning, but project-based learning is a wider and longer process, a multidisciplinary one. Moreover, it is structured way of working. Problem-based learning is subject-related, while project-based learning is based on real-life problems and in mathematics with mathematical modeling. Both (problem and project-based learning) are considered "21st century education" because they promote 21st century skills: creativity. self-confidence, problem solving and cooperation.

Therefore, STEAM education is based on project based and problem-based learning.

4.6 Inquiry based learning

In this learning method students are very involved in the learning process, and they are focused on questions, critical thinking and problem-solving. Here all the questions have definite answers and solutions. Teachers provide students with support and assistance as they lead them to discover the answers to the problems posed. With IBL for STEAM teaching, students are encouraged to think about and individualize learning as opposed to memorizing principles or hiding in groups, as can be done with other methods (CcHUB, 2020).

In an authentic STEAM lesson, reaching knowledge is not smooth. Questions are set to the students, and failure is reframed as part of the learning process. Goals, decisions, and solutions are generated by the students within the limitations of their learning context (access to materials and tools or achieving curriculum standards). Students control their own investigations. When the inquiry results in a product that provides a solution to a real problem, the inquiry is authentically STEAM.

5. STEAM technology

Since technology is integrate part of STEAM (Science Technology, Engineering, Arts and Mathematics) it is obvious that STEAM education is based on resent high level technology. STEAM education increases digital competences of teachers and student and contributes to the development of highly technological societies. This means that STEAM education is important both from an individual and social perspective. In fact, by increasing the opportunities and challenges in education and work-life, it contributes the high-level technology development of the society.
The *GeoGebra* package is a free and an open-source dynamic software created by Markus Hohenwarter for all levels of education. It is a dynamic mathematics software, supporting science, technology, engineering, arts and mathematics (STEAM) education and innovations in teaching and learning worldwide (https://www.geogebra.org/about). *GeoGebra* enables simultaneous dynamic

multiple representations, which means simultaneous algebraic, verbal, and graphical representations of mathematical objects. There is an algebraic, 3 different graphical views, and dynamic changes from one view cause simultaneous changes in other views.

The main characteristic of GeoGebra, is its simplicity of use and low teacher training demands. This is a very powerful dynamic software, with wide application possibilities. Research has shown that in this way, students are better acquainted and accustomed to a formal mathematical language by associating it with graphic representation and interpretation of a real problem in a dynamic environment. This improves the learning process and creates a positive attitude towards mathematics. There are also reports about the positive results of application of *GeoGebra* in teaching and learning sciences, mathematics and especially calculus (Takaci et.all 2015; Sekulic, et all 2020).

Videoconference is another key ingredient in online learning—and we are lucky that video conferencing today is so affordable and accessible! Teaching through video conferencing is actually very challenging, as changing media means changing the communication patterns, and good teachers are most often poor video actors. In video conferencing, we have a poor attention curve, interactions are slow, there are many distractors (we are not ‘at school’), and the overall timing should be different in order to be effective.

Video conferencing can be a powerful tool to engage in prepared live interaction situations (Q&A, feedback, project reviews, etc.) but can hardly replace classroom teaching when this means lecturing. It is actually more flexible than being in class: we can have short sessions with small groups (e.g. for project review), talk for a few minutes with one student who has a question or ask students to work in groups, etc., but it is hardly effective with a larger group for longer periods. If by ‘tools-based approach’ you mean online as entirely through ‘live teaching’, then I think the danger is that teaching staff will burn out, students will be overloaded, and opportunities will be missed for students to engage in valuable learning activities (synchronous, asynchronous, online, offline) that do not need a strong/live teacher presence. In other words, (combining questions 3 and 4) I think there is strong evidence to say that students benefit from a mix of activities—some of which depend on live teaching and teacher-led discussion and some of which do not need a live teacher presence—set tasks, using pre-prepared materials and/or student-generated materials, etc.

Covid19 crisis speed up the development of Web possibilities, such as Skype, Viber, TEAMS, Vebex, Zoom, which enable discussions among learning participants, where students can express their thoughts and understand each other. Besides that, there are lot of program packages such as GeoGebra, Maple, Mathematica, Sindarella and other. In school education the most popular and most used is dynamic package GeoGebra.

In the following, we shall present the technology integrated in the previous methodology such as computer supporting collaborative learning, Mathematical modeling in computer environment, as well as computer-based project and problem learning. Also, we shall present package GeoGebra.

5.1 Constructism in computer environment

Technology is no longer a luxury. People are used to fast paced, multi-sensory, highly interactive comfort and convenience all around them. Constructivists recognize the fact that learners require opportunities to assimilate new information in repetitive and multiple ways. Computers can serve as coaches by locating the problem and allowing for as much rehearsal, practice and help as necessary to accomplish the task. The use of computers can enhance cognitive powers of students during thinking, problem solving and learning. We are inundated with technology pervading every aspect of our lives.

Technology is increasingly being touted as an optimal medium for the application of constructivist principles to learning. Numerous online environments and technology-based projects are showing that theory can effectively guide educational practice. Software can be used in constructivist ways whereby students can design and create artwork, explore simulations, problem-solve in multimedia presentations, experiment in virtual worlds, participate in musical creations, investigate web sites, or robotic constructions.

4.2 Computer supported collaborative learning

Today we have very much developed a branch of collaborative learning named Computer supported collaborative learning and it is analyzed in lot of research. According to Stahl, et all (2006, p.1) “Computer-supported collaborative learning (CSCL) is an emerging branch of the learning science connected with studying how people learn together with the help of computers (Takaci, at all, 2015).

Computer supported collaborative learning contributes to constructivist learning and therefore it is important to work on it improving in the sense of STEAM.

4.3 Mathematical modeling in computer environment

Mathematical modeling process is the most successful when it is done in the computer environment because it allows making connections between algebraic, graphic, and verbal representations and manipulation with all three of them by using dynamic properties. Much research is done in computer technology in modeling, analyzing their contribution to creating a better model and understanding of the relationship between the mathematical model and the real world (Sekulić et all, 2020).

Using *GeoGebra* for working with real life situations contributes to a better realization of mathematical modeling activities by giving students opportunities for visualization of problems, for exploration of its elements and making relationships and connections between real and mathematical world.

6 Blended learning in STEAM education

What emphasizes STEAM from the traditional science and math education is the blended learning environment. This method teaches students computational thinking and focuses on the real-world applications of problem solving (Elaine J. H, 2014). STEAM education begins while students are very young:

- Elementary school – the STEAM curses are basic; the goal is to pique students' interest into them wanting to pursue the courses.

- Middle school – the curses become more rigorous and challenging; student exploration of STEAM related careers begins at this level.
- High school - the curses are still rigorous and challenging; students are preparing for post-secondary education and employment.

Blended learning is a teaching and a learning process that combine face-to- face instructions with computer-mediated instruction (Graham, 2013, Anthony 2020). Blended learning is the combination of different didactic approaches (cooperative learning, discovery learning expository, presentations, etc.) and delivery methods (personal communication, broadcasting, publishing, etc.) (Graham, 2013).

Blended learning as a combination of distance learning and face-to-face learning in a computing environment has spread rapidly following the rapid growth of new education technologies. If the teacher and students are geographically separated, then we have distance learning while online or e-learning is being considered in a new technological environment. The condition for distance learning is a strong Internet and the "World Wide Web", and therefore it is online or e-learning. But face-to-face learning is also often applied in a computer environment (Anthony et all, 2020, Graham, 2013).

The basic philosophy of online learning is to deliver the right materials, to the right people at the right time and in the right place in the right amount and in the right context using the most appropriate medium. Online learning is the way in which it is performed and is based on the use of modern computer and communication technology, where special emphasis is placed on interactivity and adapting learning to the needs of the individual. Teaching materials processed through online learning require transformation in relation to teaching materials used in traditional teaching (Anthony et all, 2020, Graham, 2013).

Online learning, in relation to different ways of presenting content, media format and content delivery methods, can be divided into two categories: asynchronous and synchronous. As their names suggest, these two categories of online learning are characterized by the nature of the interaction between teacher and student. In the asynchronous way of learning, the role of the teacher is static. The teacher posts information on a web page that the student can access at any time after that.

The student and the teacher do not have to be online at the same time. The interaction between teachers and students comes down to, say, filling out workbooks by the student who then teaches to the teacher or to an online knowledge test whereby they are the results of the student's knowledge test are available only after the teacher reviews and evaluates the test. The interaction between the participants of the teaching process is done through e-mail, debate groups and forums and mentoring through asking questions and giving answers. Asynchronous learning respects individual differences students and allows students with greater prior knowledge to progress faster. This kind of work reduces costs and increases the economy of the teaching process.

7 The importance of STEAM education

According to a report by Elaine J. H. (2014), there is the need for 8.65 million workers in STEAM-related jobs such as: computing, traditional engineering, physical sciences, life sciences and mathematics. STEAM jobs do not all require higher education or even a college degree. Less than half of entry-level STEAM jobs require a bachelor's degree or higher. However, a four-year degree is incredibly helpful with salary — the average advertised starting salary for entry-level STEAM jobs with a bachelor's requirement was 26 percent higher than jobs in the non-STEAM fields.

The STEAM is also important because

- students learn across contexts,
- early exposure increases passion,
- for the careers,
- STEAM is fun.

8 STEAM Teaching Practices

In this part are given some aptitudes and proficiencies that seem to be valuable for the STEAM teachers (Jolly A., 2013):

1. Believe in your students.
2. Transfer control of the learning process to the students - Develop new roles and rules that stress student responsibility.
3. Foster curiosity - Pose problems rather than answers and send students on a search for solutions.
4. Provide hands-on, experiential learning - Learning through reflection and doing is compelling.
5. Increase collaboration among students - Get comfortable with teamwork.
6. Accept failure - accept failure that accompanies taking a risk and experimenting, knowing that they might not get it right.
7. Be an inspiring leader and role model for your students.
8. Accept some drawbacks.
9. Evolve and grow as a learner - Develop your skills in facilitating so that students focus on learning how to think like engineers.
10. Learn in community - Work with your colleagues to study effective ways of teaching STEAM lessons.

8.1 Creating an inclusive STEAM environment

Below are created a list of suggestions for how to create an inclusive and diverse learning environment for the students in the classroom:

- Promote a range of STEAM role models and heroes, including different genders and ethnicities. Organize talks from these people or show your students inspiring videos.
- Try to create educational programs or clubs for underrepresented groups in STEAM, such as a girl's STEAM club or engineering club.
- Create an inclusive and welcoming environment for both genders.
- Connect students with STEAM mentors. These could be older students who can encourage them or help them with projects, or STEAM volunteers from outside of school who have careers within STEAM fields. (FutureLearn, 2021)

8.2 The eight essential elements of inclusive STEAM high schools

According to LaForce, M. et al., 2016 and 2019 the eight elements include six core elements (four instructional and two non-instructional) as well as two supporting elements. Each element is composed of several strategies representing the concrete ways that members of inclusive STEAM schools and their communities work to reach the schools' goals. Instructional elements are focused on pedagogical strategies and achieving academic goals for students, unlike noninstructional. Noninstructional elements contain strategies that do not directly focus on achieving academic goals for students. These non-instructional elements represent schools' intentions to facilitate student social and emotional outcomes, as well as to make STEAM education system better. Supporting elements include strategies and external factors that support both Instructional and non-instructional elements. Each of the eight elements is described in more detail below.

Instructional elements

1. **Personalization of Learning**

The Personalization of Learning Element refers to the fact that each student learns differently according to their capability, desires, and interests. Examples of components that comprise the Personalization of Learning Element include "teacher differentiation of instruction based on learning needs," "flexible schedule," and "student autonomy."

2. **Problem-Based Learning (PBL)** – this element is described previously.

3. **Rigorous Learning**

Rigorous Learning refers to processes that are challenging for the students and expected from them to learn at high levels. Examples of components that comprise the Rigorous Learning Element include “teacher facilitation of students engaging in real-world content,” “students engage in cognitively demanding work,” and “core course sequence.”

4. **Career, Technology, and Life Skills**

This element entails instruction and learning experiences that focus on student skills and knowledge that they will use in their careers and life. Examples of components included in this element include “students use technology,” “students participate in career-readiness activities” and “student-led demonstrations of learning.”

Non-instructional elements

5. **School community and belonging**

This element includes generating a strong school culture of professionalism and help students easier to accept the emotional needs. Some examples of components that comprise this element include “staff support the needs of the whole student,” “students treat each other with trust and respect,” “students contribute to school decision-making,” and “staff emphasize on a school code of behavior and values.”

6. **External Community**

External Community represents schools’ efforts and commitment to establishing and maintaining relationships with community members and institutions and sharing best practices and strategies with other schools. Examples of components that comprise this element include “school establishes and maintains a community presence,” “staff spread practices,” and “students participate in service learning.”

Supporting elements

7. **Staff Foundations**

Staff Foundations are components that "set the stage" for student-directed components to take place. To implement instructional and non-instructional components, schools in our sample report that certain foundations must be in place. For example, "staff collaboration" and "school leader supports staff growth and development" can set the foundation for components such as "teacher facilitation of interdisciplinary connections" to occur. Other examples of components included in this element include "staff participate in decision-making" and "common planning time."

8. **Essential Factors**

This element refers to group of conditions that school stakeholders identified as key to the successful implementation of their schools. factors are defined as components external to the school model, the external climate (political, community) and characteristics of the user. For example, some schools described "family involvement" as an important support for enacting the school's mission. Other schools described key staff attitudes (such as a "belief that all students can learn") as a necessary support to reaching school goals. Other components in essential factors include "staff are flexible and open to change" and designation as a "regional school."

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