Co-funded by the Erasmus+ Programme of the European Union

PROJECT TITLE: **Mathematics of the Future: Understanding and Application of Mathematics with the help of Technology, FutureMath**

Programme: Erasmus+

Key Action: Cooperation for innovation and the exchange of good practices

Action Type: Strategic Partnerships for higher education

Ref. No.: 2020-1-RS01-KA203-065388

Intellectual Output 1: **Analysis report on state of art in using technologies to support teaching in Mathematics after Covid-19 crisis**

Result: **Suggested Calculus topics on which the new teaching methodology will be developed**

Prepared by UNS, BMU, UPT, SIM, GDU,

Topics plans

The topics plan for the lessons in STEAM sense, based on presented previous STEAM principles explored within the "Collection of good practices and current STEAM methods and distance learning before and during Covid-19 has the following form:

TOPIC PLAN		
Partner organization		
Topic		
Lesson title		
Learning objectives		**Methodology** ☐Modeling ☐Collaborative learning ☐Project based learning ☐Problem based learning **Strategies/Activities** ☐Graphic Organizer ☐Think/Pair/Share ☐Discussion questions **Assessment for learning** ☐Observations ☐Conversations ☐Work sample ☐Conference ☐Check list ☐Diagnostics
Aim of the lecture / Description of the practical problem		
Previous knowledge assumed:		
Introduction / Theoretical basics		

Action		
Materials / equipment / digital tools / software		**Assessment as learning** ☐Self-assessment ☐Peer-assessment ☐Presentation ☐Graphic Organizer ☐Homework **Assessment of learning** ☐Test ☐Quiz ☐Presentation ☐Project ☐Published work
Consolidation		
Reflections and next steps		
Activities that worked	**Parts to be revisited**	
References		

In preparation Topics in STEAM sense first, it is necessary to give the learning objectives of the lecture. Then the aim of the lecture together with the description of the practical and other subject problems connecting with this topics should be given.

The students' previous knowledge for learning this topics should be predicted in order to enable student construvistic way of learning.

At the beginning of the lecture the Introduction, containing theoretical basics, and approach should be exposed together with the action.

Materials: equipment, digital tools, software need to be prepared.

At the end the Consolidation, as well as reflections and next steps: Activities that worked and

Parts to be revised should be prepared after class.

Also, the following STEAM approach needs to be prepared in adwance:

- **Methodology:** Modeling, or (and) Collaborative learning, or (and) Project based learning, or (and) Problem based learning.
- **Strategies-Activities:** Graphic Organizer, or (and) Think/Pair/Share or (and) Discussion questions
- **Assessment for learning:** Observations, or (and) Conversations, or (and) Work sample, or (and) Conference, Check list, or (and) Diagnostics
- **Assessment as learning:** Self-assessment**,** or (and) Peer-assessment**,** or (and) Presentation**,** or (and) Graphic Organizer**,** or (and) Homework
- **Assessment of learning:** Test**,** or (and) Quiz**,** or (and) Presentation**,** or (and) Project**,** or (and) Published work

References should be put at the end.

STEAM methodology is developed within IO2.
Assessment for learning should present feedback that students can use to improve their performance. This could be Observations, or (and) Conversations, or (and) Work sample, or (and) Conference, Check list, or (and) Diagnostics.
Assessment as learning is a process where teachers permit students to think about and monitor their own learning and develop internal feedback through Self-assessment**,** or (and) Peer-assessment**,** or (and) Presentation**,** or (and) Graphic Organizer**,** or (and) Homework.
Assessment of learning Assessment of learning address to procedures (Test**,** or (and) Quiz**,** or (and) Presentation**,** or (and) Project**,** or (and) Published work) designed to confirm what students knowledge.

Selected Calculus topics for applying STEAM principles with the Futuremath project activities

The selected calculus topics for applying STEAM principles and responsible partners are:

1. Sequence and series of numbers and function---responsible-**UPT**
2. Real functions --responsible- **UNS**
3. Limits of Function ---responsible-**BMU**
4. Applications of the derivatives (tangent line, normal line, extremal problems) ---responsible-**GDU**
5. Indefinite and definite integral, with applications ---responsible-**UNS**
6. Improper integral ---responsible-**UPT**

7. Functions with two and more variables ---responsible-**BMU**
8. Differential equations (models with differential equations) ---responsible-**GDU**
9. Multiple integrals ---responsible-**UPT**
10. Line and Curvilinear integrals ---responsible- **SIM**
11. Surface integrals ---responsible- **SIM**
12. Complex numbers ---responsible-**UNS**.

Teaching material of these topics, by using STEAM technology and methodology, will be prepared in accordance with the tasks of FUTUREMATH project.

Example of Topisc plans for Definite integral in Steam principles.

<table>
<tr><th colspan="3">TOPIC PLAN</th></tr>
<tr><td>Partner organization</td><td colspan="2">UNS</td></tr>
<tr><td>Topic</td><td colspan="2">Definite integral</td></tr>
<tr><td>Lesson title</td><td colspan="2">The introduction of definite integral</td></tr>
<tr><td>Learning objectives</td><td>Work with definite integrals and its application for determining the area of plane objects.</td><td rowspan="2">Methodology
xModeling
☐Collaborative learning
☐Project based learning
xProblem based learning

Strategies/Activities
☐Graphic Organizer
☐Think/Pair/Share
xDiscussion questions

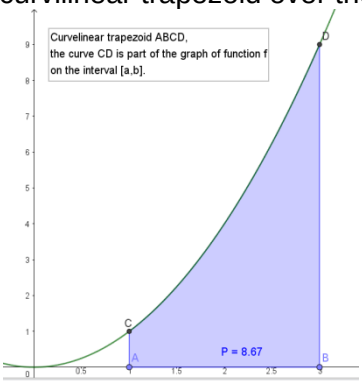
Assessment for learning
xObservations
xConversations
xWork sample</td></tr>
<tr><td>Aim of the lecture / Description of the practical problem</td><td>The aim of the lecture is the evaluation of area curvilinear trapezoid over the interval [a, b].
 

Fig.1</td></tr>
</table>

Previous knowledge assumed:	Derivatives and antiderivaties, their calculations and applications	☐Conference ☐Check list ☐Diagnostics **Assessment as learning** ☐Self-assessment ☐Peer-assessment ☐Presentation ☐Graphic Organizer xHomework **Assessment of learning** xTest ☐Quiz ☐Presentation ☐Project ☐Published work
Introduction / Theoretical basics	Let the continuous function f is given on the interval $[a,b]$. Let us divide the interval $[a,b]$ on n subintervals such that $a = x_0 < x_1 < x_2 < x_3 < \cdots < x_{n-1} < x_n = b.$ Let us denote the length of $i-$ interval with $\Delta x_i = x_i - x_{i-1}, \quad i = 1,2,\dots,n,$ $\Delta x = \max_i \Delta x_i.$ Let c_i be the points from $[x_{i-1}x_i]$, $i = 1,2,\dots,n.$ If the limit $\lim_{\substack{n\to\infty \\ \Delta x\to 0}} \sum_{i=1}^{n} f(c_i)\,\Delta x_i$ exists for each division of interval $[a,b]$ and every choice of n points $c_i \in [x_{i-1}x_i]$, then it defines the definite or Riman integral of the function f on the interval $[a,b]$, i.e., $\lim_{\substack{n\to\infty \\ \Delta x\to 0}} \sum_{i=1}^{n} f(c_i)\,\Delta x_i := \int_a^b f(x)dx,$ 1) If $f(x) > 0$, then $\int_a^b f(x)dx$, represent the area of curvilinear trapezoid under the graph of f over the interval [0, 1]. 2) If $f(x) < 0$, then $\left\lvert \int_a^b f(x)dx \right\rvert$ represent the area of curvilinear trapezoid under the graph of f over the interval [0, 1].	

Action	Questions to students: 3) The function $f(x) = x^2$, is given. Determine the area of the inscribed and prescribed rectangle in, and over, curvilinear trapezoid over the interval [0, 1] 4) The function $f(x) = x^2$, is given. The interval [0,1] is divided in 4 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 5) The function $f(x) = x^2$, is given. The interval [0,1] is divided in 5 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 6) The function $f(x) = x^2$, is given. The interval [0,1] is divided in 10 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 7) Analyze the obtained results	
Materials / equipment / digital tools / software	*The materials* are given in the references given at the end from this topic plan; *Equipment*: classroom, green board; *Digital tools*: laptop, projector; *Software: GeoGebra*, used for multiple representation of presented object	
Consolidation	• The teachers and the students use: teaching materials, equipment, digital tools, *GeoGebra* software; • The teacher's and students' discussion about the cognitive conflicts that appear; • Independent solving of simple tasks by the students under the supervision of the teacher; • Given of examples by the teacher for introducing a new concept in a cooperation and a discussion with the students; • Assignment of homework by the teacher with a time limit until the next class.	
Reflections and next steps		

Activities that worked	Parts to be revisited
The teacher should fulfilled this part after the class	The definition of definite integral and its application for determining the area, will be revised, after the overview of the students' homework and discussion at the beginning of the next class, in the form that should be necessary.

References

1) Bittinger, M. L., Ellenbogen, D. J., Surgent, S.A., (2012)"Calculus and its applications", Addison-Wesley.
2) Schmeelk, J., Takaci, D., Takaci, A., (2013) Elementary analysis through examples and exercises, Kluwer, Springer Science & Business Media.
3) Stewart J., (2006) Calculus, Thomson Learning, China.
4) Takači, Dj., Stankov, G., Milanovic, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups,. *Computers and Education, Vol. 82*, 421-431
5) The film Definition of definite integral can be found on the platform **https://cloud.pmf.uns.ac.rs/s/pQXwNsPD3GtcyEZ**