Co-funded by the Erasmus+ Programme of the European Union

PROJECT TITLE: **Mathematics of the Future: Understanding and Application of Mathematics with the help of Technology, FutureMath**

Programme:	Erasmus+
Key Action:	Cooperation for innovation and the exchange of good practices
Action Type:	Strategic Partnerships for higher education
Ref. No.:	2020-1-RS01-KA203-065388

Intellectual Output 1: **Analysis report on state of art in using technologies to support teaching in Mathematics after Covid-19 crisis**

Result: **Suggested Calculus topics on which the new teaching methodology will be developed**

Prepared by UNS, BMU, UPT, SIM, GDU,

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<table>
<tr><th colspan="3">TOPIC PLAN</th></tr>
<tr><td>Partner organization</td><td colspan="2">UNS</td></tr>
<tr><td>Topic</td><td colspan="2">Definite integral</td></tr>
<tr><td>Lesson title</td><td colspan="2">The introduction of definite integral</td></tr>
<tr><td>Learning objectives</td><td>Work with definite integrals and its application for determining the area of plane objects.</td><td rowspan="2">Methodology
xModeling
☐Collaborative learning
☐Project based learning
xProblem based learning

Strategies/Activities
☐Graphic Organizer
☐Think/Pair/Share
xDiscussion questions

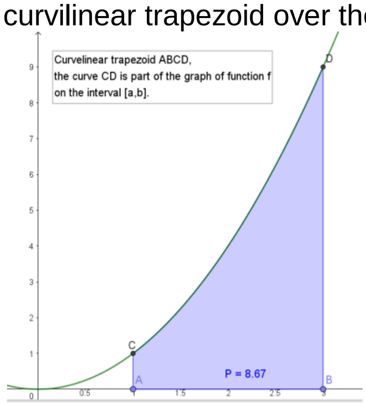
Assessment for learning
xObservations
xConversations
xWork sample</td></tr>
<tr><td>Aim of the lecture / Description of the practical problem</td><td>The aim of the lecture is the evaluation of area curvilinear trapezoid over the interval [a, b].


Fig.1</td></tr>
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<td>Previous knowledge assumed:</td>
<td>Derivatives and antiderivaties, their calculations and applications</td>
<td rowspan="2">☐Conference
☐Check list
☐Diagnostics

Assessment as learning
☐Self-assessment
☐Peer-assessment
☐Presentation
☐Graphic Organizer
xHomework

Assessment of learning
xTest
☐Quiz
☐Presentation
☐Project
☐Published work</td>
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<td>Introduction / Theoretical basics</td>
<td>

Let the continuous function f is given on the interval $[a, b]$. Let us divide the interval $[a, b]$ on n subintervals such that

$$a = x_0 < x_1 < x_2 < x_3 < \cdots < x_{n-1} < x_n = b.$$

Let us denote the length of $i -$ interval with

$$\Delta\, x_i = x_i - x_{i-1}, \qquad i = 1,2, \dots, n,$$

$$\Delta x = \max_i \Delta\, x_i .$$

Let c_i be the points from $[x_{i-1} x_i]$, $i = 1,2, \dots, n$. If the limit

$$\lim_{\substack{n\to\infty \\ \Delta x \to 0}} \sum_{i=1}^{n} f(c_i)\, \Delta\, x_i$$

exists for each division of interval $[a, b]$ and every choice of n points $c_i \in [x_{i-1} x_i]$, then it defines the definite or Riman integral of the function f on the interval $[a, b]$, i.e.,

$$\lim_{\substack{n\to\infty \\ \Delta x \to 0}} \sum_{i=1}^{n} f(c_i)\, \Delta\, x_i := \int_a^b f(x)dx,$$

1) If $f(x) > 0$, then $\int_a^b f(x)dx$, represent the area of curvilinear trapezoid under the graph of f over the interval [0, 1].
2) If $f(x) < 0$, then $\left|\int_a^b f(x)dx\right|$ represent the area of curvilinear trapezoid under the graph of f over the interval [0, 1].

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Action	Questions to students: 3) The function $f(x) = x^2,$ is given. Determine the area of the inscribed and prescribed rectangle in, and over, curvilinear trapezoid over the interval [0, 1] 4) The function $f(x) = x^2,$ is given. The interval [0,1] is divided in 4 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 5) The function $f(x) = x^2,$ is given. The interval [0,1] is divided in 5 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 6) The function $f(x) = x^2,$ is given. The interval [0,1] is divided in 10 subintervals Determine the area of the sum of inscribed and sum of prescribed rectangles in, and over, curvilinear trapezoid over the subinterval of interval [0,1]. 7) Analyze the obtained results	
Materials / equipment / digital tools / software	*The materials* are given in the references given at the end from this topic plan; *Equipment*: classroom, green board; *Digital tools*: laptop, projector; *Software: GeoGebra*, used for multiple representation of presented object	
Consolidation	• The teachers and the students use: teaching materials, equipment, digital tools, *GeoGebra* software; • The teacher's and students' discussion about the cognitive conflicts that appear; • Independent solving of simple tasks by the students under the supervision of the teacher; • Given of examples by the teacher for introducing a new concept in a cooperation and a discussion with the students; • Assignment of homework by the teacher with a time limit until the next class.	
Reflections and next steps		

Activities that worked	**Parts to be revisited**
The teacher should fulfilled this part after the class	The definition of definite integral and its application for determining the area, will be revised, after the overview of the students' homework and discussion at the beginning of the next class, in the form that should be necessary.

References

1) Bittinger, M. L., Ellenbogen, D. J., Surgent, S.A., (2012)"Calculus and its applications", Addison-Wesley.
2) Schmeelk, J., Takaci, D., Takaci, A., (2013) Elementary analysis through examples and exercises, Kluwer, Springer Science & Business Media.
3) Stewart J., (2006) Calculus, Thomson Learning, China.
4) Takači, Dj., Stankov, G., Milanovic, I. (2015). Efficiency of learning environment using GeoGebra when calculus contents are learned in collaborative groups,. *Computers and Education, Vol. 82*, 421-431
5) The film Definition of definite integral can be found on the platform **https://cloud.pmf.uns.ac.rs/s/pQXwNsPD3GtcyEZ**